เฉลยแบบฝึกหัดบทที่ 4

4.1 ใช้วิธีดูลิตเติลอย่างย่อเพื่อหาผลเฉลยของระบบสมการ

$$\left.\begin{aligned} 3\hat{\beta}_1 + 2\hat{\beta}_2 + \hat{\beta}_3 &= 7 \\ 2\hat{\beta}_1 + 2\hat{\beta}_2 + \hat{\beta}_3 &= 5 \\ \hat{\beta}_1 + \hat{\beta}_2 + 4\hat{\beta}_3 &= -1 \end{aligned}\right\} \longrightarrow \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ -1 \end{bmatrix}$$

โดยวิธีการดูลิตเติลอย่างย่อ

Row	C_1	C_2	C_3	C_0	E_1	E_2	E_3	Check
R_1'	3	2	1	7	1	0	0	14
R_2'	(2)	2	1	5	0	1	0	11
R_3'	(1)	(1)	4	-1	0	0	1	6
R_1	3	(2)	[1]	7	(1)	0	0	14
r_1	1	2/3	1/3	7/3	1/3	0	0	14/3
R_2	*	2/3	[1/3]	1/3	(-2/3)	(1)	0	5/3
r_2	*	1	1/2	1/2	-1	3/2	0	5/2
R_3	*	*	7/2	-7/2	(0)	(-1/2)	(1)	1/2
r_3	*	*	1	-1	0	-1/7	2/7	1/7
B_1					1	-1	0	
B_2						11/7	-1/7	
B_3							2/7	

จาก r_3: $\hat{\beta}_3 = -1$

จาก r_2: $\hat{\beta}_2 + (1/2)\hat{\beta}_3 = 1/2$ ดังนั้น $\hat{\beta}_2 = (1/2) + (1/2) = 1$

จาก r_1: $\hat{\beta}_1 + (2/3)\hat{\beta}_2 + (1/3)\hat{\beta}_3 = 7/3$

ดังนั้น $\hat{\beta}_1 = (7/3) - (2/3) + (1/3) = 6/3 = 2$

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix}^{-1} = (1/7)\begin{bmatrix} 7 & -7 & 0 \\ -7 & 11 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

4.2 ทำซ้ำข้อ 4.1 สำหรับระบบสมการ

$$\begin{aligned} 2\hat{\beta}_1 + \hat{\beta}_2 + 3\hat{\beta}_3 - \hat{\beta}_4 &= 8 \\ \hat{\beta}_1 + \hat{\beta}_2 - \hat{\beta}_3 + \hat{\beta}_4 &= -1 \\ 3\hat{\beta}_1 - \hat{\beta}_2 + 3\hat{\beta}_3 - 2\hat{\beta}_4 &= 2 \\ -\hat{\beta}_1 + \hat{\beta}_2 - 2\hat{\beta}_3 + 4\hat{\beta}_4 &= 1 \end{aligned}$$

$$\longrightarrow \begin{bmatrix} 2 & 1 & 3 & -1 \\ 1 & 1 & -1 & 1 \\ 3 & -1 & 3 & -2 \\ -1 & 1 & -2 & 4 \end{bmatrix}\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 2 \\ 1 \end{bmatrix}$$

จากตารางซึ่งใช้วิธีดูลิตเติลอย่างย่อ

จาก r_4: $\hat{\beta}_4 = 1$

จาก r_3: $\hat{\beta}_3 - (1/2)\hat{\beta}_4 = 35/14$

ดังนั้น $\hat{\beta}_3 = (35/14) + (1/2) = 3$

จาก r_2: $\hat{\beta}_2 - 5\hat{\beta}_3 + 3\hat{\beta}_4 = -10$

ดังนั้น $\hat{\beta}_2 = -10 + 15 - 3 = 2$

จาก r_1: $\hat{\beta}_1 + (1/2)\hat{\beta}_2 + (3/2)\hat{\beta}_3 - (1/2)\hat{\beta}_4 = 4$

ดังนั้น $\hat{\beta}_1 = 4 - 1 - (9/2) + (1/2) = -1$

Row	C_1	C_2	C_3	C_4	C_0	E_1	E_2	E_3	E_4	Check (sum)
R_1'	2	1	3	-1	8	1	0	0	0	14
R_2'	(1)	1	-1	1	-1	0	1	0	0	2
R_3'	(3)	(-1)	3	-2	2	0	0	1	0	6
R_4'	(-1)	(1)	-2)	4	1	0	0	0	1	4
R_1	2	(1)	[3]	(-1)	8	(1)	0	0	0	14
r_1	1	1/2	3/2	-1/2	4	1/2	0	0	0	7
R_2	*	1/2	[5/2]	(3/2)	-5	(-1/2)	(1)	0	0	-5
r_2	*	1	-5	3	-10	-1	2	0	0	-10
R_3	*	*	-14	(7)	-35	(-4)	(5)	(1)	0	-40
r_3	*	*	1	-1/2	35/14	2/7	-5/14	-1/14	0	20/7
R_4	*	*	*	5/2	5/2	(0)	(-1/2)	(1/2)	(1)	6
r_4	*	*	*	1	1	0	-1/5	1/5	2/5	12/5
B_1						-1/7	3/7	2/7	0	
B_2							11/35	-16/35	-1/5	
B_3								1/35	1/5	
B_4									2/5	

$$\beta' = [-1 \quad 2 \quad 3 \quad 1]$$

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 1 & 1 & -1 & 1 \\ 3 & -1 & 3 & -2 \\ -1 & 1 & -2 & 4 \end{bmatrix}^{-1} = (1/35) \begin{bmatrix} -5 & 15 & 10 & 0 \\ 15 & 11 & -16 & -7 \\ 10 & -16 & 1 & 7 \\ 0 & -7 & 7 & 14 \end{bmatrix}$$

4.3 จากตัวอย่างที่ 3.4

$$X'X = \begin{bmatrix} 8 & 49 & 120 \\ 49 & 555 & 1014 \\ 120 & 1014 & 2110 \end{bmatrix}, \quad X'Y = \begin{bmatrix} 290 \\ 3264.9 \\ 5967.2 \end{bmatrix}$$

Row	C_1	C_2	C_3	C_0	E_1	E_2	E_3	Check(sum)
R_1'	8	49	120	290	1	0	0	468
R_2'	(49)	555	1014	3264.9	0	1	0	4883.9
R_3'	(120)	(1014)	2110	5967.2	0	0	1	9212.2
R_1	8	(49)	120	290	1	0	0	468
r_1	1	6.125	15	36.25	0.125	0	0	58.5
R_2	*	254.875	(279)	1488.65	-6.125	1	0	2017.4
r_2	*	1	1.0946542	5.8407062	-0.0240313	0.0039234	0	7.9152525
R_3	*	*	(4.5914782)	-12.3570298	-8.2952673	-1.0946286	1	-16.155448
r_3	*	*	1	-2.6912966	-1.8066659	-0.2384043	0.2177947	-3.5185721
B_1					15.2589682	1.9535965	-1.8066652	
B_2						0.2648875	-0.2384043	
B_3							0.2177497	

1) จาก r_3: $\hat{\beta}_2 = -2.6912966 = -2.69$

จาก r_2: $\hat{\beta}_1 + 1.0946542\hat{\beta}_2 = 5.8407062$, $\hat{\beta}_1 = 8.7867453 = 8.79$

จาก r_1: $\hat{\beta}_0 + 6.125\hat{\beta}_1 + 15\hat{\beta}_2 = 36.25$, $\hat{\beta}_0 = 22.8006$

ดังนั้น $\hat{\beta}' = [22.80 \quad 8.79 \quad -2.69]$

$$R(\beta) = \sum_{i=1}^{3} R_{i0}r_{i0} = (290)(36.25)+(1488.65)(5.8407062)$$
$$+(-12.3570298)(-2.6912966)$$
$$= 19240.5237168$$

$$\hat{\sigma}^2 = [Y'Y - R(\beta)]/(n - 3)$$
$$= (19286.9 - 19240.5237)/5$$
$$= 9.2754$$

2) $D^* = \hat{\beta}_0 + \hat{\beta}_1 T_1 + \hat{\beta}_2 T_2$

$= 22.8 + 8.79T_1 - 2.69T_2$

3) ทดสอบ H_0: $\beta_1 = \beta_2 = 0$ และ H_0: $\beta = 0$

ให้ $\beta = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$

$$R(\gamma_1) = R(\beta_0) = R_{10}r_{10}$$
$$= 290(36.25) = 10512.5$$

$$R(\gamma_2 | \gamma_1) = \sum_{i=2}^{3} R_{i0}r_{i0}$$

$$= (1488.65)(5.8407062)+(-12.3570298)(-2.6912966)$$
$$= 8728.0237$$

ANOVA H_o: β = o , H_o: β_1 = β = 0

S.V.	d.f.	SS	MS	f_c
Due to β	**3**	**19240.5237**	**6413.5076**	**691.45****
Due to β_o(unadj.)	**1**	10512.5		
Due to β_1, β_2(adj.)	**2**	8728.0237	4364.0118	**470.49****
Error	**5**	46.377	**9.2754**	
Total (uncorrected)	8	**19286.9**		

$f_{(3,5)..01}$ = 12.06, $f_{(2,5),01}$ = **13.27**