

ภาคผนวก ก
ตัวอย่างโปรแกรมภาษาฟอร์แทรน
สำหรับการวิเคราะห์เชิงตัวเลข

บทที่ 2

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C*****RNR*****      ตัวอย่างการเรียกใช้ Subroutine NEWTON RAPHSON
      EXTERNAL F,FP
10  WRITE(6,1)
1   FORMAT(1X,'ENTER NUMSIG,MAXIT,X0(INITIAL GUESS) ')
      READ(5,*)NUMSIG,MAXIT,X0
      CALL NR(F,FP,NUMSIG,MAXIT,X0,X,Y)
      WRITE(6,2)
2   FORMAT(1X,'ENTER 1 TO CONTINUE, 2 TO STOP ')
      READ(5,*)IFLAG
      IF(IFLAG.EQ.1)GO TO 10
      STOP
      END
C*****FUNCTION F(X):f(x)*****
C*****f(x) from the example in Section 2.1C*****
      FUNCTION F(X)
      F=EXP(-X)-COS(X)
      RETURN
      END
C*****FUNCTION FP(X):First derivative of f(x) = f'(x)*****
      FUNCTION FP(X)
      FP=SIN(X)-EXP(-X)
      RETURN
      END
C*****SUBROUTINE NEWTON RAPHSON*****
C*****See Slope Method Algorithm in Section 2.1A*****
      SUBROUTINE NR(F,FP,NUMSIG,MAXIT,X0,X,Y)
      RELTOL=10.**(-NUMSIG)
      XPREV=X0
      YPREV=F(X0)
      WRITE(6,6)XPREV,YPREV
6   FORMAT(1X,'K=0',I2,' X0=',F12.7,' Y0=',E13.7)
      DO 10 K=1,MAXIT
      SLOPE=FP(XPREV)
      DELTAX=-YPREV/SLOPE
      X=XPREV+DELTAX
      Y=F(X)
      WRITE(6,2)K,X,Y,DELTAX
2   FORMAT(1X,'K=',I2,' X =',F12.7,' Y =',E13.7,
t   ' DELTAX= ',E14.7)
      XPREV=X
      YPREV=Y
      IF(ABS(DELTAX).LE.RELTOL*ABS(X))GO TO 9
10  CONTINUE
      WRITE(6,4)MAXIT
4   FORMAT(1X,'DESIRED ACCURACY IS NOT EVIDENT IN',I3,
+ ' ITERATIONS')
      RETURN
9   WRITE(6,3)X,NUMSIG
3   FORMAT(/1X,'X= ',F12.7,' APPROXIMATES REQUIRED ROOT TO ',I2,
+ ' SIGNIFICANT DIGITS')
      RETURN
      END
```

ตัวอย่าง output จากการเรียกใช้ Subroutine NEWTON RAPHSON

สำหรับตัวอย่างในหัวข้อ 2.1C

ENTER NUMSIG,MAXIT,XO(INITIAL GUESS) UNIT 5? **con**
7 10 1.3

K= 0 X0= 1.3000000 Y0= , 5032927E-02
K= 1 X = 1.2927170 Y = .1442432E-04 DELTAX= -.7283263E-02
K= 2 X = 1.2926960 Y = .2980232E-07 DELTAX= -.2099426E-04
K= 3 **x = 1.2926960** Y = .2980232E-07 DELTAX= -.4337729E-07

X= 1.2926960 APPROXIMATES REQUIRED ROOT TO 7 SIGNIFICANT DIGITS
ENTER 3 TO CONTINUE, 2 TO STOP 1

ENTER NUMSIG,MAXIT,XO(INITIAL GUESS) 7 10 4.712389

K= 0 **x01- 4.7123870** Y0- .8983279E-02
K= 1 X = **4.1212920** Y = .2663583E-06 DELTAX= .8903298E-02
K= 2 X = **4.7212930** Y = -.2151355E-06 DELTAX= .2640180E-06

X= 4.7212930 APPROXIMATES REQUIRED ROOT TO 7 SIGNIFICANT DIGITS
ENTER 1 TO CONTINUE, 2 TO STOP 2
Stop - Program terminated.

ตัวอย่างการเรียกใช้ subroutine BAIRSTOW

```

C*****INTERACTIVE CALLING PROGRAM FOR BAIRSTOW'S METHOD*****
  DIMENSION A(10),B(10),C(10)
  MAXIT=20
  WRITE(6,1)
1  FORMAT(1X,'ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)')
  READ(5,*)N,(A(I),I=1,N+1)
  M=N+1
  WRITE(6,2)
2  FORMAT(1X,'ENTER DESIRED # OF SIGNIFICANT DIGITS')
  READ(5,*)NUMSIG
  WRITE(6,3)
3  FORMAT(1X,'ENTER INITIAL R,INITIAL S')
  READ(5,*)RO,SO
  CALL BAIRSTOW(A,B,C,M,RO,SO,NUMSIG,MAXIT)
  STOP
  END

C*****SUBROUTINE FOR BAIRSTOW'S METHOD*****
C*****See Bairstow's Method Algorithm in Section 2.2C*****
  SUBROUTINE BAIRSTOW(A,B,C,M,RO,SO,NUMSIG,MAXIT)
C*****TO FIND A QUADRATIC FACTOR q(x)=x2-rx-s OF AN Nth DEGREE*****
C*****POLYNOMIAL p(x)*****
C*****M=N+1*****
  DIMENSION A(M),B(M),C(M)
  N=M-1
  B(1)=A(1)
  C(1)=B(1)
  R=RO
  S=SO
  WRITE(6,5)
5  FORMAT(3X,'k',7X,'dr',12X,'ds',14X,'r',12X,'s')
  WRITE(6,6)R,S
6  FORMAT(3X,'0',8X,'-',13X,'-',7X,2F13.7)
  DO 11 K=1,MAXIT
  B(2)=A(2)+B(1)*R
  C(2)=B(2)+C(1)*R
  DO 22 I=3,M
  B(I)=A(I)+R*B(I-1)+C*I*(I-2)
  C(I)=B(I)+R*C(I-1)+S*C(I-2)
22  CONTINUE
  DET=C(N)*C(N-2)-C(N-1)*C(N-1)
  DR=(B(N)*C(N-1)-B(N+1)*C(N-2))/DET
  DS=(B(N+1)*C(N-1)-B(N)*C(N))/DET
  R=R+DR
  S=S+DS
  WRITE(6,1)K,DR,DS,R,S
11  FORMAT(14,2E15.7,2F13.7)
  IF (ABS(DR) .LE. 10.**(-NUMSIG)*AMAX1(1.,ABS(R)) .AND.
+ ABS(DS) .LE. 10.**(-NUMSIG)*AMAX1(1.,ABS(S)))GO TO 9
9  CONTINUE
  WRITE(6,2)R,S
2  FORMAT(1X,'q(X)=X*X-',1',F12.7,')*X-',1',F12.7,')')
  WRITE(6,3)(B(I),I=1,N-1)
3  FORMAT(1X,'COEFFICIENTS OF Q(X):'
+5X,5F12.7/5X,5F12.7/5X,5F12.7)
  RETURN
  END

```

ตัวอย่างการเรียกใช้ Subroutine LINFIT

```

C*****CALLING PROGRAM FOR LINFIT*****
C*****PROBLEM 5.3 OR 205*****
      DIMENSION X(6),Y(6),G(6),H(6)
      REAL LNY(6),LNX(6)
      DATA X,Y/1.,2.,3.,4.,5.,6.,2.3,6.1,10.7,16.0,21.9,28.3/
      OPEN(6,FILE='B:RLINFIT1.OUT',STATUS='NEW')
      N=6
      DO 10 I=1,6
      LNY(I)=ALOG(Y(I))
10     LNX(I)=ALOG(X(I))
      CALL LINFIT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL LINFIT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      ALPHA=EXP(YCEPT)
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
11     FORMAT(1X,'ALPHA = ',F10.5,' BETA = ',F10.5)
      EG=0
      DO 20 I=1,6
      G(I)=ALPHA*EXP(BETA*X(I))
20     EG=EG+(G(I)-Y(I))**2
      WRITE(6,12)EG
12     FORMAT(1X,'E[g(x)]=',F12.7)
      CALL LINFIT(N,LNX,LNY,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,LNX,LNY,SQUERR,YCEPT,SLOPE)
      ALPHA=EXP(YCEPT)
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
      EH=0
      DO 30 I=1,6
      H(I)=ALPHA*X(I)**BETA
30     EH=EH+(H(I)-Y(I))**2
      WRITE(6,13)EH
13     FORMAT(1X,'E[h(x)]=',E12.7)
      STOP
      END
C*****SUBROUTINE TO PRINT OUTPUT*****
      SUBROUTINE PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      DIMENSION X(20),Y(20)
      WRITE(6,2)(X(I),I=1,N)
2     FORMAT(/' X: ',6F10.5)
      WRITE(6,3)(Y(I),I=1,N)
3     FORMAT(' Y: ',6F10.5)
      WRITE(6,4)SQUERR,YCEPT,SLOPE
4     FORMAT(' ERROR SUM OF SQUARE E[L ] = ',E14.7/
+           ' Y - INTERCEPT           = ',E14.7/
+           ' SLOPE                       = ',E14.7)
      WRITE(6,5)YCEPT,SLOPE
5     FORMAT(' PREDICTION EQUATION: Y = ',F8.5,' + ',F8.5,' *X ')
      RETURN
      END

```

ตัวอย่าง Output จากการเรียกใช้ Subroutine LINFIT สำหรับแบบฝึกหัดข้อ 5.3

X: 1.00000 2.00000 3.00000 4.00000 5.00000 6.00000
 Y: 2.30000 6.10000 10.70000 16.00000 21.90000 28.30000
 ERROR SUM OF SQUARE E[L] = .3961578E+01
 Y - INTERCEPT = -.4053333E+01
 SLOPE = .5220000E+01
 PREDICTION EQUATION: $Y = -4.05333 + 5.22000 * X$

X: 1.00000 2.00000 3.00000 4.00000 5.00000 6.00000
 Y: .83291 1.80829 2.37024 2.77259 3.08649 3.34286
 ERROR SUM OF SQUARE E[L] = .2744317E+00
 Y - INTERCEPT = .6902263E+00
 SLOPE = .4796201E+00
 PREDICTION EQUATION: $Y = .69023 + .47962 * X$
 ALPHA = 1.99417 BETA = .47962
 $E[g(x)] = .6379251E+02$

X: .00000 .69315 1.09861 1.38629 1.60944 1.79176
 Y: .83291 1.80829 2.37024 2.77259 3.08649 3.34286
 ERROR SUM OF SQUARE E[L] = .2519730E-04
 Y - INTERCEPT = .8342940E+00
 SLOPE = .1399493E+01
 PREDICTION EQUATION: $Y = .83429 + 1.39949 * X$
 ALPHA = 2.30319 BETA = 1.39949
 $E[h(x)] = .2546936E-02$

บทที่ 6

ตัวอย่างการเรียกใช้ FUNCTION POFZ

```

C*****RPOFZ*****
C*****INTERACTIVE CALLING PROGRAM FOR FUNCTION POFZ*****
      DIMENSION X(20),Y(20)
      WRITE(6,1)
1      FORMAT(1X,'ENTER NUMBER OF POINT-; ')
      READ(5,*)NP
      WRITE(6,11)
11     FORMAT(JX,'ENTER X(I)''S 'I
      READ(5,*)(X(I),I=1,NP)
      WRITE(6,33)
33     FORMAT(1X,'ENTER Y(I)''S ')
      READ(5,*)(Y(I),I=1,NP)
      WRITE(6,6)(X(I),I=1,NP)
6      FORMAT(1X,'X(I): ',10F10.3)
      WRITE(6,7)(Y(I),I=1,NP)
7      FORMAT(1X,'Y(I): ',10F10.3)
12     WRITE(6,2)
2      FORMAT(1X,'ENTER K,M,Z FOR p(K,K+M) OF Z ')
      READ(5,*)K,M,Z
      YY=POFZ(X,Y,NP,K,M,Z)
      KM=K+M
      WRITE(6,3)K,KM,Z,YY
3      FORMAT(1X,'p('',12.',',12.') OF '.F10.3,' ',10.3)
      WRITE(o.4)
4      FORMAT(1X,'ENTER 1 TO CONTINUE, 2 TO STOP')
      READ(5,*)IFLAG
      IF(IFLAG .EQ. 1)GO TO 12
      STOP
      END
C*****FUNCTION FOR LAGRANGE INTERPOLATION *****
C*****See Lagrange Interpolation Algorithm*****
C*****in Section 6.1C *****
      FUNCTION POFZ(X,Y,NP,K,M,Z)
C*****THIS FUNCTION EVALUATES THE LAGRANGE FORM OF*****
C*****P[K,K+M] OF (Z)*****
      DIMENSION X(NP),Y(NP)
      POFZ=0
      DO 10 J=K,K+M
          TERMJ=Y(J)
          DO 11 I=K,K+M
              IF(I .NE. J)THEN
                  TERMJ=TERMJ*(Z-X(I))/(X(J)-X(I))
              ENDIF
11         CONTINUE
          POFZ=POFZ+TERMJ
10     CONTINUE
      RETURN
      END

```

ตัวอย่าง Output จากการเรียกใช้ Subroutine POFZ

สำหรับตัวอย่างในหัวข้อ 6.1D

UNIT 5? con

ENTER NUMBER OF POINTS UNIT 5? con

5

ENTER X(I)'S -2 0 1 4 5

ENTER Y(I)'S -8 0 1 64 125

X(I): -2.000 .000 1.000 4.000 5.000

Y(I): -8.000 .000 1.000 64.000 125.000

ENTER K,M,Z FOR p(K,K+M) OF Z 2 2 3

p(2, 4) OF 3.000= 33.000

ENTER 1 TO CONTINUE, 2 TO STOP. 1

ENTER K,M,Z FOR p(K,K+M) OF Z 2 1 3 2

p(1, 4) OF 2.000= 8.000

ENTER 1 TO CONTINUE, 2 TO STOP. 1

ENTER K,M,Z FOR p(K,K+M) OF Z 1 4 2

p(1, 5) OF 2.000= 8.000

ENTER 1 TO CONTINUE, 2 TO STOP.

ตัวอย่างการเรียกใช้ Subroutine FORMDD

```

C*****RFORMDD*****
C*****CALLING PROGRAM FOR SUBROUTINE FORMDD*****
  DIMENSION X(10),Y(10),DD(10,10)
  WRITE(6,1)
1  FORMAT(1X,'ENTER NUMBER OF POINTS (LESS THAN 10) ')
  READ(5,*)NP
  WRITE(6,2)
2  FORMAT(1X,'ENTER X'S : ')
  READ(5,*)(X(I),I=1,NP)
  WRITE(6,3)
3  FORMAT(1X,'ENTER Y'S : ')
  READ(5,*)(Y(I),I=1,NP)
  WRITE(6,4)(X(I),I=1,NP)
4  FORMAT(1X,'X(I): ',5F10.3)
  WRITE(6,5)(Y(I),I=1,NP)
5  FORMAT(1X,'Y(I): ',5F10.3)
  CALL FORMDD(X,Y,NP,DD)
  STOP
  END
C*****FORMING A DIVIDED DIFFERENCE TABLE DD*****
C*****See FORMDD ALGORITHM in Section 6.2B*****
  SUBROUTINE FORMDD(X,Y,N,DD)
  DIMENSION X(N),Y(N),DD(N,N)
  DO 10 K=1,N
10  DD(K,1)=Y(K)
  DO 12 M=1,N-1
  DO 13 K=1,N-M
  DD(K,M+1)=(DD(K+1,M)-DD(K,M))/(X(K+M)-X(K))
13  WRITE(6,1)K,M+1,DD(K,M+1),DD(K,M+1)
1  FORMAT(1X,'DD(',I2,',',I2,',')=',F12.7,F14.7)
12  CONTINUE
  RETURN
  END

```

ตัวอย่าง Output จากการใช้ Subroutine FORMDD

ENTER NUMBER OF POINTS (LESS THAN 10) UNIT 5? con

6

ENTER X'S : 0 .2 .4 .6 .8 1.0

ENTER Y'S : .5 .5793 .6554 .7257 .7881 .8413

X(I):	.000	.200	.400	.600	.800
X(I):	1.000				
Y(I):	.500	.579	.655	.726	.788
Y(I):	.841				
DD(1, 2) =	.3964999	.3964999E+00			
DD(2, 2) =	.3805000	.3805000E+00			
DD(3, 2) =	.3515002	.3515002E+00			
DD(4, 2) =	.3119999	.3119999E+00			
DD(5, 2) =	.2660001	.2660001E+00			
DD(1, 3) =	-.0399999	-.3999999E-01			
DD(2, 3) =	-.0724994	-.7249943E-01			
DD(3, 3) =	-.0987506	-.9875059E-01			
DD(4, 3) =	-.1149997	-.1149997E+00			
DD(1, 4) =	-.0541658	-.5416583E-01			
DD(2, 4) =	-.0437519	-.4375193E-01			
DD(3, 4) =	-.0270819	-.2708188E-01			
DD(1, 5) =	.0130174	.1301737E-01			
DD(2, 5) =	.0208376	.2083757E-01			
DD(1, 6) =	.0078202	.7820197E-02	Stop	Program terminated.	

บทที่ 7

```

C*****ROMBERG***** ตัวอย่างการเรียกใช้ Subroutine ROMBERG
C*****CALLING PROGRAM FOR SUBROUTINE ROMBERG*****
      DIMENSION X(100),Y(100),T(20,20)
      EXTERNAL FUNC
5      WRITE(6,1)
1      FORMAT(1X,'ENTER a,b,N,MAXROWS,NUMSIG ')
      READ(5,*)A,B,N,MAXROWS,NUMSIG
      CALL ROMBERG(A,B,N,MAXROWS,NUMSIG,FUNC,X,Y,T)
      WRITE(6,2)
2      FORMAT(1X,'ENTER 1 TO CONTINUE,2 TO STOP ')
      READ(5,*)IFLAG
      IF(IFLAG .EQ. 1)GO TO 5
      STOP
      END
C*****FUNCTION TO BE INTEGRATED*****
      FUNCTION FUNC(X)
C*****THIS FUNCTION IS TO BE CHANGED*****
      FUNC=ALOG(X)
      RETURN
      END
C*****ROMBERG INTEGRATION*****
C*****See ROMBERG INTEGRATION ALGORITHM in Section 7.40*****
      SUBROUTINE ROMBERG(A,B,N,MAXROWS,NUMSIG,FUNC,X,Y,T)
      DIMENSION X(100),Y(100),T(MAXROWS,MAXROWS)
      EXTERNAL FUNC
      H=(B-A)/N
      CALL SUMOE(X,Y,N,A,B,H,SUMO,SUME,FUNC)
      SUM=SUMO+SUME
      T(1,1)=H*((FUNC(A)+FUNC(B))/2.+SUM)
      WRITE(6,9)T(1,1)
9      FORMAT(1X,'T( 1, 1)= ',F12.6)
      RELTOL=10**(-NUMSIG)
      DO 19 K=2, MAXROWS
      H=H/2
      N=2*N
      CALL SUMOE(X,Y,N,A,B,H,SUMO,SUME,FUNC)
      T(K,1)=.5*(T(K-1,1)+2*H*SUMO)
      WRITE(6,7)K,T(K,1)
7      FORMAT(1X,'T( ',I2, ', 1)= ',F12.6)
      DO 18 I=2,K
      T(K,I)=(4**(I-1)*T(K,I-1) - T(K-1,I-1))/(4**(I-1)-1)
18      WRITE(6,8)K,I,T(K,I)
8      FORMAT(1X,'T( ',I2, ', ',I2, ')= ',F12.6)
      IF(ABS(T(K,K)-T(K,K-1)) .LE. RELTOL*ABS(T(K,K)))GO TO 99
19      CONTINUE
      WRITE(6,2)MAXROWS
2      FORMAT(1X,I2,' ITERATIONS DID NOT YIELD THE DESIRED ',
+ 'ACCURACY.')
      RETURN
99      WRITE(6,3)K,K,T(K,K)
3      FORMAT(1X,'ESTIMATED INTEGRAL IS T( ',I2, ', ',I2, ') ',F12.6)
      RETURN
      END

```

```

C*****SUMOE*****
C*****Points in ROMBERG ALGORITHM:y(0),y(1),y(2),...,y(n-1),y(n)*****
C*****SUMD = y(1) t y(3) t ...
C*****SUME = y(2) t y(4) t ...
C*****Points in this subroutine :y(1), y(2), y(3), ,y( n) ,y(n+1)*****
SUBROUTINE SUMOE(X,Y,N,A,B,H,SUMD,SUME,FUNC)
DIMENSION X(N),Y(N)
SUMD=0
SUME=0
DO 10 I=2,N
X(I)=A+(I-1)*H
10 Y(I)=FUNC(X(I))
DO 11 I=2,N,2
11 SUMD=SUMD+Y(I)
DO 12 I=3,N,2
12 SUME=SUME+Y(I)
RE TURN
END

```

ตัวอย่าง Output จากการใช้ Subroutine ROMBERG

```

C:\FORTRAN>romberg
File name missing or blank - Please enter name
UNIT 6? con
ENTER a,b,N,MAXROWS,NUMSIG UNIT 5? con
1 2.2 3 5 5
T( 1, 1)= .527395
T( 2, 1)= .532792
T( 2, 2)= .534591
T( 3, 1)= .534152
T( 3, 2)= .534605
T( 3, 3)= .534606
T( 4, 1)= .534493
T( 4, 2)= .534606
T( 4, 3)= .534606
T( 4, 4)= .534606
ESTIMATED INTEGRAL IS T( 4, 4) .534606
ENTER 1 TO CONTINUE,2 TO STOP

```

```

C*****PR713A*****
C*****Problem 7.13a*****
C*****3-point GAUSS QUADRATURE for integral of SQRT(X)*****
C*****over [0,4]*****
      DIMENSION XI(3),X(3),GAMMA(3)
      FUNC(X)=SQRT(X)
      DATA A,B/0.0,4.0/
C*****GAUSS SAMPLE POINTS (XI's)*****
C*****and GAUSS SAMPLE WEIGHTS (GAMMA's)*****
C*****from TABLE 7.5-1*****
      XI(1)=SQRT(.6)
      XI(2)=-XI(1)
      XI(3)=0
      GAMMA(1)=5./9.
      GAMMA(2)=GAMMA(1)
      GAMMA(3)=8./9.
      OPEN(6,FILE='B:PR713A.OUT',STATUS='NEW')
      WRITE(6,5)
5      FORMAT(1X,'Problem 7.13a, 3-Point Gauss Quadrature'/)
      DO 11 I=1,3
11     X(I)=A+(B-A)/2*(XI(I)+1)
      PINT=0
      DO 10 I=1,3
      Y=FUNC(X(I))
      Z=GAMMA(I)*Y
      WRITE(6,1)I,X(I),Y,Z
1      FORMAT(1X,'X(',I1,')=' ,F12.7,' F(X)=' ,F12.7,
+ ' GAMMA(I)*F(X(I))=' ,F12.7)
      LO PINT=PINT+Z
      PINT=(B-A)/2.*PINT
      WRITE(6,2)PINT
2      FORMAT(/1X,'ESTIMATED DEFINITE INTEGRAL = ',F12.7)
      STOP
      END
      Problem 7.13a, 3-Point Gauss Quadrature

X(1)= 3.5471530 F(X)= 1.8839300 GAMMA(I)*F(X(I))= 1.0466280
X(2)= .4508066 F(X)= .6714213 GAMMA(I)*F(X(I))= .3730119
X(3)= 2.0000000 F(X)= 1.4142140 GAMMA(I)*F(X(I))= 1.2570790

ESTIMATED DEFINITE INTEGRAL = 5.3534370

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C*****PR713B*****
C*****Problem 7.13b*****
C*****5-point GAUSS QUADRATURE for INTEGRAL of SQRT(X)*****
C*****over [0,4]*****
      DIMENSION XI(5),X(5),GAMMA(5)
      FUNC(X)=SQRT(X)
      DATA A,B/0.0,4.0/
C*****GAUSS SAMPLE POINTS (XI's)*****
C*****and GAUSS SAMPLE WEIGHTS (GAMMA's)*****
C*****from TABLE 7.5-1*****
      DATA XI/-.9061798,.9061798,-.5384693,.5384693,0./
      DATA GAMMA/.2369269,.2369269,.4786287,.4786287,.5688889/
      OPEN(6,FILE='B:PR713B.OUT',STATUS='NEW')
      WRITE(6,5)
5      FORMAT(1X,'Problem 7.13b, 5-Point Gauss Quadrature'/)
      DO 11 I=1,5
11     X(I)=A+(B-A)/2*(XI(I)+1)
      PINT=0
      DO 10 I=1,5
      Y=FUNC(X(I))
      Z=GAMMA(I)*Y
      WRITE(6,1)I,X(I),Y,Z
1      FORMAT(1X,'X(',I1,')=',F12.7,' F(X)=',F12.7,
t' GAMMA(I)*F(X(I))=',F12.7)
      LO PINT=PINT+Z
      PINT=(B-A)/2.*PINT
      WRITE(6,2)PINT
2      FORMAT(/1X,'ESTIMATED DEFINITE INTEGRAL = ',F12.7)
      STOP
      END

```

Problem 7.13b, 5-Point Gauss Quadrature

X(1)=	.1876404	F(X)=	.4331748	GAMMA(I)*F(X(I))=	.1026308
X(2)=	3.8123600	F(X)=	1.9525260	GAMMA(I)*F(X(I))=	.4626060
X(3)=	.9230614	F(X)=	.9607608	GAMMA(I)*F(X(I))=	.4598477
X(4)=	3.0769390	F(X)=	1.7541200	GAMMA(I)*F(X(I))=	.8395724
X(5)=	2.0000000	F(X)=	1.4142140	GAMMA(I)*F(X(I))=	.8045304

ESTIMATED DEFINITE INTEGRAL = 5.3383750

บทที่ 8

ตัวอย่างการเรียกใช้ Subroutine RK4

```
C*****RRK4*****
C*****CALLING PROGRAM FOR SUBROUTINE RK4*****
C*****RK4 : 4-TH ORDER RUNGE-KUTTA METHOD*****
      DOUBLE PRECISION YO
      EXTERNAL FTEST
      WRITE(6,1)
1     FORMAT(1X,'INPUT TO,TF,YO ')
      READ(5,*)TO,TF,YO
      WRITE(6,2)
2     FORMAT(1X,'INPUT # STEPS NPRINT ')
      READ(5,*)NSTEPS,NPRINT
      WRITE(6,3)
3     FORMAT(6X,'T',12X,'Y')
      CALL RK4(FTEST,TO,TF,NSTEPS,NPRINT,YO)
      STOP
      END

C*****FUNCTIONIDN FTEST*****
C*****This FUNCTION is to be changed upon IVP*****
      FUNCTION FTEST(T,Y)
      DOUBLE PRECISION Y
C*****Example in Section 8.2C*****
      FTEST=-T*Y*Y
      RETURN
      END

C*****SUBROUTINE RK4*****
      SUBROUTINE RK4(F,T,TF,NSTEPS,NPRINT,Y)
      REAL M1,M2,M3,M4
      DOUBLE PRECISION Y
C*****
C     THIS SUBROUTINE INTEGRATES FROM TO TO TF THE 1ST ORDER IVP
C     Y' = F(T,Y)    Y(TO)=YO (INITIAL T,Y)
C     USING NSTEPS STEPS OF THE 4TH ORDER RUNGE-KUTTA METHOD.
C     IF NPRINT>0, IT PRINTS T AND Y EVERY NPRINT STEPS.
C     NOTE: F MUST BE DECLARED EXTERNALLY IN THE CALLING PROGRAM
C*****
      IF (NPRINT .GT. 0)WRITE(6,1)T,Y
1     FORMAT(F10.3,3X,D14.7)
      H=(TF-T)/NSTEPS
      DO 10 J=1,NSTEPS
          M1=F(T,Y)
          M2=F(T+0.5*H,Y+0.5*H*M1)
          M3=F(T+0.5*H,Y+0.5*H*M2)
          M4=F(T+H,Y+H*M3)
          T=T+H
          Y=Y+H*(M1+2*(M2+M3)+M4)/6
          IF(NPRINT .GT. 0 .AND. MOD(J,NPRINT) .EQ. 0)
t      WRITE(6,1)T,Y
10    CONTINUE
      RETURN
      END
```

```

C*****RRK43*****
C*****CALLING PROGRAM FOR SUBROUTINE RK4*****
C*****RK4 : 4-TH ORDER RUNGE-KUTTA METHOD*****
      DOUBLE PRECISION YO
      EXTERNAL FTEST
      WRITE(6,1)
1     FORMAT(1X,'INPUT TO,TF,YO ')
      READ(5,*)TO,TF,YO
      WRITE(6,2)
2     FORMAT(1X,'INPUT # STEPS NPRINT ')
      READ(5,*)NSTEPS,NPRINT
      WRITE(6,5)
5     FORMAT(7X,'Output for Problem 8.6b')
      WRITE(6,3)
3     FORMAT(7X,'T',12X,'Y')
      CALL RK4(FTEST,TO,TF,NSTEPS,NPRINT,YO)
      STOP
      END
C*****FUNCTION FTEST*****
C*****This FUNCTION is to be changed upon IVP*****
      FUNCTION FTEST(T,Y)
      DOUBLE PRECISION Y
C*****IVP from Problem 8.6b*****
      FTEST=.5*(Y/T+T/Y)
      RETURN
      END
C*****SUBROUTINE RK4*****
      SUBROUTINE RK4(F,T,TF,NSTEPS,NPRINT,Y)
      REAL M1,M2,M3,M4
      DOUBLE PRECISION Y
C*****
C     THIS SUBROUTINE INTEGRATES FROM TO TO TF THE 1ST ORDER IVP
C      $Y' = F(T,Y)$   $Y(TO)=YO$  (INITIAL T,Y)
C     USING NSTEPS STEPS OF THE 4TH ORDER RUNGE-KUTTA METHOD.
C     IF NPRINT>0, IT PRINTS T AND Y EVERY NPRINT STEPS.
C     NOTE: F MUST BE DECLARED EXTERNALLY IN THE CALLING PROGRAM
C*****
      IF (NPRINT .GT. 0)WRITE(6,1)T,Y
1     FORMAT(F10.3,3X,D14.7)
      H=(TF-T)/NSTEPS
      DO 10 J=1,NSTEPS
          M1=F(T,Y)
          M2=F(T+0.5*H,Y+0.5*H*M1)
          M3=F(T+0.5*H,Y+0.5*H*M2)
          M4=F(T+H,Y+H*M3)
          T=T+H
          Y=Y+H*(M1+2*(M2+M3)+M4)/6
          IF(NPRINT .GT. 0 .AND. MOD(J,NPRINT) .EQ. 0)
+           WRITE(6,1)T,Y
10    CONTINUE
      RETURN
      END

```


ตัวอย่าง Output จากการเรียกใช้ Subroutine RK4
 สำหรับ IVP จากตัวอย่างในหัวข้อ 8 . x

```
C:\FORTRAN>rrk4
File name missing or blank - Please enter name
UNIT 6? con

INPUT TO,TF,YO UNIT 5? con
2 3 1

INPUT # STEPS NPRINT 10 1

      T           Y
2.000      .1000000D+01
2.100      .8298852D+00
2.200      .7042368D+00
2.300      .6079135D+00
2.400      .5319244D+00
2.500      .4705964D+00
2.600      .4201750D+00
2.700      .3780778D+00
2.800      .3424708D+00
2.900      .3120168D+00
3.000      .2857180D+00Stop Program terminated.
```

ตัวอย่าง Output จากการเรียกใช้ Subroutine RK4
 สำหรับ IVP จากแบบฝึกหัดข้อ 8.6b

```
C:\FORTRAN>rrk43
File name missing or blank * Please enter name
UNIT 6'1' con

INPUT TO,TF,YO UNIT 5? con
1 3 3

INPUT # STEPS NPRINT 10 1

      Output for Problem 8.6b
      T           Y
1.000      .3000000D+01
1.200      .3322651D+01
1.400      .3627674D+01
1.600      .3919187D+01
1.800      .4200004D+01
2.000      .4472140D+01
2.200      .4737092D+01
2.400      .4996003D+01
2.600      .5249767D+01
2.800      .5499096D+01
3.000      .5744568D+01Stop Program terminated.
```

ตัวอย่างการเรียกใช้ Subroutine RKF4

```

C*****RKF4*****
C*****CALLING PROGRAM FOR SUBROUTINE RKF4*****
C*****RKF4 : 4 TH ORDER RUNGE-KUTTA-FEHLBERG METHOD*****
      DOUBLE PRECISION YO(10)
      COMMON N, TO, TF, YO
      DATA SCAMIN, SCAMAX, RMAX/0.1, 4.0, 1.E-4/
      WRITE(6,1)
1     FORMAT(1X,'INPUT N, TO ,TF ,NPRINT ')
      READ(5,*)N, TO, TF, NPRINT
      WRITE(6,2)N
2     FORMAT(1X,'INPUT ',12,' COMPONENTS OF YO ')
      READ(5,*)(YO(I),I=1,N)
      WRITE(6,3)TO, TF, NPRINT,
3     FORMAT(1X,'INTEGRATING FROM',F8.4,' TO',F8.4,
+           ' PRINTING EVERY',15,' STEPS '//5X,
+           'H',6X,'T',10X,'H',8X,'-----Y----->')
      CALL RKF4(RMAX, SCAMIN, SCAMAX, NPRINT)
      STOP
      END

C*****SUBROUTINE EVALF*****
      SUBROUTINE EVALF(T,Y,F)
      DIMENSION F(1)
      DOUBLE PRECISION Y(1)
C*****F(1) is to be changed upon IVP*****
C*****Example in Section 8.2C*****
      F(1) = -T*Y(1)*Y(1)
*
      RETURN
      END

C*****SUBROUTINE SUMK*****
      SUBROUTINE SUMK(J, P, Q1, Q2, Q3, Q4, Q5)
      REAL K(10,6), F(10)
      DOUBLE PRECISION Y(10), SUM(10)
      COMMON N, T, TF, Y
      COMMON /KCALC/ H, K
C*****
C      THIS SUBROUTINE EVALUATES THE N-VECTOR
C
C          SUM = Y + Q1*ROW1(K) + ... + Q5*ROW5(K)
C
C      IF J = 0, IT SETS Y = SUM.
C      IF 1 <= J <= 6, IT PUTS F(T+P*H,SUM) IN JTH COLUMN OF K
C*****
      DO 10 I = 1, N
          SUM(I) = Y(I) + DBLE(Q1)*K(I,1) + DBLE(Q2)*K(I,2) +
+          DBLE(Q3)*K(I,3) + DBLE(Q4)*K(I,4) + DBLE(Q5)*K(I,5)
          IF(J .EQ. 0)Y(I) = SUM(I)
10     CONTINUE
          IF(J .EQ. 0)RETURN
          CALL EVALF(T+P*H, SUM, F)
          DO 20 I = 1,N
              K(I,J) = H*F(I)
20     CONTINUE
      RETURN
      END

```

```

C*****SUBROUTINE RKF4*****
      SUBROUTINE RKF4(RMAX,SCAMIN,SCAMAX,NPRINT)
      REAL K(10,6)
      DOUBLE PRECISION Y(10)
      COMMON N, T, TF, Y
      COMMON /KCALC/ H, K
C*****
C      THIS SUBROUTINE USES THE 4TH ORDER RUNGE-KUTTA-FEHLBERG
C      ALGORITHM TO INTEGRATE N (N < 11) COUPLED FIRST ORDER IVPs.
C      I.E.      Y' = F(T,Y)   Y(T0) = Y0   (T0, Y0 ARE INITIAL T, Y)
C      FROM T0 TO TF, WHERE TF CAN BE TO THE LEFT OF T0.
C      VALUES OF T, H, AND VECTOR Y ARE PRINTED EVERY NPRINT ITERATIONS.
C      IF NPRINT = 0, NOTHING IS PRINTED (UNLESS H BECOMES TOO SMALL).
C*****
C      INITIALIZE STEPSIZE, MINIMUM ALLOWABLE STEPSIZE, AND COUNTER:
      H = SIGN(RMAX*.25,TF-T)
      HMIN=0.5E-4*H
      ITER=0
      IF(NPRINT .GT. 0) WRITE(6,2)ITER,T,H,(Y(I),I=1,N)
2      FORMAT(I6,F9.4,11E15.7)

C      ITERATE:
C*****BEGIN LOOP BY SETTING H TO TF-T IF T+H PASSES TF
10     IF(H*(1+H-|F|) .GT. 0)H=TF-T
      C
C*****PUT VECTOR KJ IN JTH COLUMN OF MATRIX K, J=1,...,6
      CALL SUMK(1,0.,0.,0.,0.,0.,0.)
      CALL SUMK(2,.25, .25, 0., 0., 0., 0.)
      CALL SUMK(3,.375, 3./32, 9./32, 0., 0., 0.)
      CALL SUMK(4,12./13, 1932./2197, -7200./2197, 7296./2197, 0., 0.)
      CALL SUMK(5,1., 439./216, -8., 3680./513, -845./4104, 0.)
      CALL SUMK(6,-5,-8./27, 2., -3544./2565, 1859./4104, -11./40)
      C
C*****FORM ERREST = ESTIMATE OF ERROR OF NEXT Y(I) FOR I = 1,...,N
C*****AND FIND RATIO = THE LARGEST OF THE ERREST/H RATIOS
      RATIO = 0.
      DO 20 I=1,N
      ERREST=K(I,1)/360-128.*K(I,3)/4275-2197.*K(I,4)/75240+
+      K(I,5)/50+2.*K(I,6)/55
      RATIO=AMAX1(RATIO,ABS(ERREST/H))
20     CONTINUE
      C
C*****TEST ACCURACY OF NEXT Y. IF OK, UPDATE T, Y AND ITER
      IF(RATIO .GT. RMAX) GO TO 30
      T=T+H
      CALL SUMK(0, 0., .25./216, 0., 1408./2565, 2197./4104, -0.2)
      ITER=ITER+1
      C
      IF(NPRINT .GT. 0 .AND. (MOD(ITER,NPRINT) .EQ. 0.OR.T.EQ.TF))
+      WRITE(6,2)ITER,T,H,(Y(I),I=1,N)

```

```

C
C*****SET SCALE(BETWEEN SCAMIN AND SCAMAX) AND UPDATE H
30  SCALE=.84*(RMAX/RATIO)**.25
    IF(SCALE .LT. SCAMIN)SCALE=SCAMIN
    IF(SCALE .GT. SCAMAX)SCALE=SCAMAX
    H=SCALE*H
C
C*****TERMINATION TESTS
    IF(T .EQ. TF)RETURN
    IF(ABS(H) .GT. HHIN)GO TO 10
C*****END OF LOOP
C
    WRITE(6,3)T
3   FORMAT(1X,'APPARENT SINGULARITY NEAR T= ',E10.3)
    RETURN
    END

```

ตัวอย่าง Output จากการเรียกใช้ Subroutine RKF4

C:\FORTRAN>rrkf4

File name missing or blank - Please enter name

UNIT 6? con

INPUT N, TO ,TF ,NPRINT UNIT 5? con

1 2 4 1

INPUT 1 COMPONENTS OF YO 1

INTEGRATING FROM 2.0000 TO 4.0000, PRINTING EVERY 1 STEPS

3	T	H	-----Y----->
0	2.0000	.1000000E+00	.1000000E+01
1	2.1000	.1000000E+00	.8298735E+00
2	2.2115	.1114867E+00	.6918773E+00
3	2.3496	.1381161E+00	.5680757E+00
4	2.5204	.1708163E+00	.4595003E+00
5	2.7342	.2137788E+00	.3652362E+00
6	3.0050	.2707816E+00	.2844945E+00
7	3.3529	.3478962E+00	.2164052E+00
8	3.8077	.4547834E+00	.1600214E+00
9	4.0000	.1923409E+00	.1428565E+00Stop - Program terminated.

บทที่ ๑

```

C*****RJACOBI      ตัวอย่างการเรียกใช้ Subroutine ROTATE
C*****CALLING PROGRAM FOR SUBROUTINE ROTATE*****
C*****SEE JACOBI'S METHOD ALGORITHM      *****
C*****IN SECTION 9.3C      *****
      DIMENSION A(5,5),U(5,5)
      COMMON N,NROTAT,A,U
      OPEN(6,FILE='B:JACOBI.OUT',STATUS='NEW')
      WRITE(7,1)
1      FORMAT(1X,'ENTER N(LESS THAN 5):',
t ' ORDER OF SYMMETRIC MATRIX A')
      READ(5,*)N
      WRITE(6,51)N
51      FORMAT(1X,'N = ',I3)
      WRITE(7,2)
2      FORMAT(1X,'ENTER N*N ELEMENTS OF A:',
t ' COLUMN BY COLUMN')
      READ(5,*)((A(I,J),I=1,N),J=1,N)
      WRITE(6,22)((A(I,J),I=1,N),J=1,N)
22      FORMAT(/1X,'A(I,J) BY COLUMN:',3F12.7,(/18X,3F12.7))
      WRITE(7,3)
3      FORMAT(1X,'ENTER NUMDEC, MAXIT ')
      READ(5,*)NUMDEC,MAXIT
      WRITE(6,77)NUMDEC,MAXIT
77      FORMAT(/1X,'NUMDEC = ',I3,' ,MAXIT = ',I3/)
      DO 10 I = 1,N
      DO 10 3 = 1,N
      IF(I .EQ. J)THEN
          U(I,J) = 1
      ELSE
          U(I,J) = 0
      ENDIF
10      CONTINUE
      ABSTOL = 10.0**(-NUMDEC)
      WRITE(6,88)ABSTOL
88      FORMAT(1X,'ABSTOL = ',F12.7/)
      NROTAT = 0
      DO 11 K = 1,MAXIT
          AMAXOD = ABS(A(1,2))
          II = 1
          JJ = 2
          DO 12 I = 1,N-1
          DO 12 J = 2,N
          IF(I .LT. J)THEN
              IF(ABS(A(I,J)) .GT. AMAXOD)THEN
                  AMAXOD = ABS(A(I,J))
                  II = I
                  JJ = J
              ENDIF
          ENDIF
12      CONTINUE
      WRITE(6,66)
66      FORMAT(/1X,' K II JJ      A(II,JJ)      AMAXOD')
      WRITE(6,6)K,II,JJ,A(II,JJ),AMAXOD
6      FORMAT(/3I3,2F12.7)

```

```

        CALL ROTATE(II, JJ)
        WRITE(6, 67)((A(I, J), I=1, N), J=1, N)
67      FORMAT(/1X, 'A(I, J) BY COLUMN:', 3F12.7, (/18X, 3F12.7))
        WRITE(6, 68)((U(I, J), I=1, N), J=1, N)
68      FORMAT(/1X, 'U(I, J) BY COLUMN:', 3F12.7, (/18X, 3F12.7))
        IF(AMAXOD .LT. ABSTOL)GO TO 99
11     CONTINUE
        WRITE(6, 4)MAXIT
4      FORMAT(/1X, 'DESIRED ACCURACY IS NOT EVIDENT IN',
+I3, ' ITERATIONS')
        STOP
99     WRITE(6, 7)NROTAT
7      FORMAT(/1X, 'NO. OF ROTATIONS PERFORMED =', I3)
        DO 13 J = 1, N
        WRITE(6, 5)A(J, J), (U(I, J), I=1, N)
5      FORMAT(/1X, 'EIGEN VALUE', F12.7/
t      1X, 'EIGEN VECTOR:', 5F12.7/)
13     CONTINUE
        STOP
        END
C*****SUBROUTINE ROTATE*****
        SUBROUTINE ROTATE(I, J)
        DIMENSION A(5, 5), U(5, 5)
        COMMON N, NROTAT, A, U
C*****
C THIS SUBROUTINE PERFORMS THE ROTATION REPLACEMENTS
C   A <-- RTRANSPOSE*A*R   AND U <-- U*R
C WHERE R = R[I, J](THETA) MAKES A(I, J) = A(J, I) = 0
C*****
        THETA = ATAN(1.0)
        IF(ABS(A(I, I)-A(J, J)) .GT. 1.E-6*ABS(A(I, I)))
t      THETA = .5*ATAN(2.*A(I, J)/(A(I, I)-A(J, J)))
        SINT = SIN(THETA)
        COST = COS(THETA)
C*****ROTATE:   A <-- RTRANS*A*R AND U <-- U*R
        DO 10 K = 1, N
            UKI = U(K, I)
            U(K, I) = UKI*COST t U(K, J)*SINT
            U(K, J) = -UKI*SINT t U(K, J)*COST
            IF(K .EQ. I .OR. K .EQ. J)GO TO 10
            A(K, I) = A(I, K)*COST t A(J, K)*SINT
            A(K, J) = -A(I, K)*SINT + A(J, K)*COST
            A(J, K) = A(K, J)
            A(I, K) = A(K, I)
10     CONTINUE
            AII = A(I, I)
            A(I, I) = AII*COST**2 t A(J, J)*SINT**2 t
+2.*A(I, J)*SINT*COST
            A(J, J) = AII*SINT**2 t A(J, J)*COST**2 -
+2.*A(I, J)*SINT*COST
            A(I, J) = 0.
            A(J, I) = 0.
            NROTAT = NROTAT+1
            RETURN
        END

```

ตัวอย่าง Output จากการใช้ Subroutine ROTATE

สำหรับ JACOBI'S METHOD ALGORITHM

N= 3

A(I,J) BY COLUMN: **3.000000** .0100000 .0200000
.0100000 **2.000000** .1000000
.0200000 .1000000 1.0000000

NUMDEC = 7 ,MAXIT = 10

ABSTOL = .0000001

K II JJ A(II,JJ) AMAXOD

1 2 3 .1000000 .1000000

A(I,J) BY COLUMN: **3.000000** .0119221 .0189173
.0119221 2.0099020 .0000000
.0189173 .0000000 .9900981

U(I,J) BY COLUMN: 1.0000000 .0000000 .0000000
.0000000 .9951333 .0985376
.0000000 -.0985376 .9951333

K II JJ A(II,JJ) AMAXOD

2 13 .0189173 .0189173

A(I,J) BY COLUMN: 3.0001780 .0119216 .0000000
.0119216 2.0099020 -.0001122
.0000000 -.0001122 .9899200

U(I,J) BY COLUMN: , 9999557 -.0009273 .0093650
.0000000 .9951333 .0985376
-.0094108 -.0985333 .9950893

K II JJ A(II,JJ) AMAXOD

3 1 2 .0119216 .0119216

A(I,J) BY COLUMN: **3.0003220** , **0000000** -.0000014
.0000000 **2.0097580** -.0001122
-.0000014 -.0001122 .9899200

U(I,J) BY COLUMN: .9998833 .0110502 .0105503
-.0120355 .9950724 .0984178
-.0094108 -.0985333 .9950893

K II JJ	A(II,JJ)	AMAXOD	
4 2 3	-.0001122	.0001122	
A(I,J) BY COLUMN:	3.0003220	.0000000	-.0000014
	.0000000	2.0097580	.0000000
	-.0000014	.0000000	.9899200
U(I,J) BY COLUMN:	.9998833	.0110502	.0105503
	-60120344	.9950833	.0983083
	-.0094121	-.0984238	.9951001

K II JJ	A(II,JJ)	AMAXOD	
5 1 3	-.0000014	.0000014	
A(I,J) BY COLUMN:	3.0003220	.0000000	.0000000
	.0000000	2.0097580	.0000000
	.0000000	.0000000	.9899200
U(I,J) BY COLUMN:	.9998833	.0110502	.0105497
	-.0120344	.9950833	.0983083
	-.0094114	-.0984238	.9951001

K II JJ	A(II,JJ)	AMAXOD	
6 1 2	.0000000	.0000000	
A(I,J) BY COLUMN:	3.0003220	.0000000	.0000000
	.0000000	2.0097580	.0000000
	.0000000	.0000000	.9899200
U(I,J) BY COLUMN:	.9998833	.0110502	.0105497
	-.0120344	.9950833	.0983083
	-.0094114	-.0984238	.9951001

NO. OF ROTATIONS PERFORMED = 6

EIGEN VALUE	3.0003220		
EIGEN VECTOR:	.9998833	.0110502	.0105497
EIGEN VALUE	2.0097580		
EIGEN VECTOR:	-.0120344	.9950833	.0983083
EIGEN VALUE	.9899200		
EIGEN VECTOR:	-.0094114	-.0984238	.9951001