เฉลยแบบฝึกหัดบทที่ ๙

9.1 ใน (a)-(b) จงหา characteristic polynomial และ eigenpairs ทั้งหมดของ A แล้วจงหา eigenpairs ของ A^{-1} ด้วย

(a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$

9.1 (a) $\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$p_A(\lambda) = \left| A - \lambda I \right| = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1$$

Characteristic Polynomial คือ

$$p_A(\lambda) = \lambda^2 - 3$$

$$p_A(\lambda) = \lambda^2 - 1 = 0$$

Eigenroots ของ A คือ $\lambda = 1 , -1$

สำหรับ $\lambda = 1$

$$Av = 1v$$

$$Av - 1v = 0$$

$$(A - I)v = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{--->} \quad \begin{aligned} -v_1 + v_2 &= 0 \\ v_1 - v_2 &= 0 \\ v_1 &= v_2 \end{aligned}$$

$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

สำหรับ $\lambda = -1$

$$Av = -1\mathbf{v}$$

$$Av + 1\mathbf{v} = 0$$

$$(A + I)\mathbf{v} = 0$$

$$\begin{bmatrix} & \\ 1 & \end{bmatrix} \mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \quad v_1 + v_2 = 0$$

$$v_1 = -v_2$$

$$v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

ดังนั้น eigenpairs ของ A คือ $(1, [1 \;\; 1]')$ และ $(-1, [1 \;\; -1]')$

eigenpairs ของ A^{-1} คือ $(1, [1 \;\; 1]')$ และ $(-1, [1 \;\; -1]')$

9.1 (b) $p_A(\lambda) = |A - \lambda I| = \begin{vmatrix} 4-\lambda & 5 \\ 3 & 2-\lambda \end{vmatrix}$

$$= (4 - \lambda)(2 - \lambda) - 15$$

$$= 8 - 6\lambda + \lambda^2 - 15$$

$$= \lambda^2 - 6\lambda - 7$$

Characteristic Polynomial คือ

$$p_A(\lambda) = \lambda^2 - 6\lambda - 7$$

$p_A(\lambda) = \lambda^2 - 6\lambda - 7 = 0$

Eigenroots ของ A คือ $\lambda = 7 , -1$

สำหรับ $\lambda = 7$

$(A - 7I)v = 0$

$$\begin{bmatrix} -3 & 5 \\ 3 & -5 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad ---> \quad -3v_1 + 5v_2 = 0$$

$$v_1 = 5v_2 / 3$$

$$v = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

สำหรับ $\lambda = -1$

$$Av = -1v$$

$$Av + 1v = 0$$

$$(A + I)v = 0$$

$$\begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad ---> \quad \begin{aligned} 5v_1 + 5v_2 &= 0 \\ 3v_1 + 3v_2 &= 0 \\ v_1 &= -v_2 \end{aligned}$$

$$v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

ดังนั้น eigenpairs ของ A คือ (7, [-5 3]') และ (-1, [1 -1]')

eigenpairs ของ A^{-1} คือ (1/7, [5 3]') และ (-1, [1 -1]')

9.2 สำหรับ A ใน (a)-(b) ของข้อ 9.1 จงหาเมตริกซ์ V ซึ่งทำให้ $V^{-1}AV = \mathrm{diag}(\lambda_1, \lambda_2)$ ถ้า V นั้นหาได้ และจงตรวจสอบว่า $V^{-1}AV = \mathrm{diag}(\lambda_1, \lambda_2)$

9.2 (a) $V = [v_1 \quad v_2] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$V^{-1} = [1/(-1-1)] \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}' = (1/-2) \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$$

$$V^{-1}AV = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \mathrm{diag}(1, -1)$$

9.2 (b) $V = [v_1 \quad v_2] = \begin{bmatrix} 5 & 1 \\ 3 & -1 \end{bmatrix}$

$$V^{-1} = [1/(-5-3)]\begin{bmatrix} -1 & -3 \\ -1 & 5 \end{bmatrix}' = (1/-8)\begin{bmatrix} -1 & -1 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/8 & 1/8 \\ 3/8 & -5/8 \end{bmatrix}$$

$$V^{-1}AV = \begin{bmatrix} 1/8 & 1/8 \\ 3/8 & -5/8 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7/8 & 7/8 \\ -3/8 & 5/8 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \text{diag}(7, -1)$$

สำหรับเมตริกซ์ A และค่าเดาเริ่มต้น x_o ในข้อ 9.3 และข้อ 9.4 จงทำ (a)-(c) ต่อไปนี้

(a) จงหา x_1, x_2, x_3, x_4 โดยใช้ Scaled Power method

(b) จงหา x_1, x_2, x_3, x_4 โดยใช้ Inverse Power Method (ใช้สูตรสำหรับ A^{-1})

(c) จงหา x_1, x_2, x_3, x_4 โดยใช้ Shifted Inverse method และใช้ shifted scalar s (ใช้สูตรสำหรับ $(A - sI)^{-1}$)

9.3

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix}, \quad x_o = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad s = 3$$

Eigenpairs: $\left[2, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right], \left[7, \begin{bmatrix} -2 \\ 3 \end{bmatrix}\right]$

9.3(a) Scaled Power Method: $Ax_k = \text{Big } X,\ x_{k+1}$

$$k = 0:\ Ax_0 = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \end{bmatrix} = (-8) \begin{bmatrix} -6/8 \\ 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ -0.75 \end{bmatrix}$$

$$k = 1:\ Ax_1 = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} -0.075 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 7.25 \end{bmatrix}$$

$$= 7.25 \begin{bmatrix} 1 \\ -0.6896551 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -0.6896551 \\ 1 \end{bmatrix}$$

$$k = 2:\ Ax_2 = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} -0.6896551 \\ 1 \end{bmatrix} = \begin{bmatrix} -4.7586204 \\ 7.0689653 \end{bmatrix}$$

$$= 7.0669653 \begin{bmatrix} -0.6731707 \\ 1 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -0.6731707 \\ 1 \end{bmatrix}$$

$$k = 3:\ Ax_3 = \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} -0.6731707 \\ 1 \end{bmatrix} = \begin{bmatrix} -4.6926828 \\ 7.0195121 \end{bmatrix}$$

$$= 7.0195121 \begin{bmatrix} \text{[illegible]}0.66851\text{[illegible]}7 \end{bmatrix}$$

9.3(b) Inverse Power Method

$$A^{-1} = \begin{bmatrix} 5/14 & 1/7 \\ 3/14 & 2/7 \end{bmatrix}$$

$$x_0 = [1\ \ -1]'$$

$$k = 0:\ A^{-1}x_0 = \begin{bmatrix} 5/14 & 1/7 \\ 3/14 & 2/7 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/14 \\ -1/14 \end{bmatrix}$$

$$= (3/14) \begin{bmatrix} 1 \\ -1/3 \end{bmatrix} = (3/14) \begin{bmatrix} 1 \\ -0.3333333 \end{bmatrix}$$

$$k = 1:\ A^{-1}x_1 = \begin{bmatrix} 3/14 & 2/7 \\ 5/14 & 1/7 \end{bmatrix} \begin{bmatrix} -1/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 13/42 \\ 5/42 \end{bmatrix}$$

$$= (13/42) \begin{bmatrix} 1 \\ 5/13 \end{bmatrix} = (13/42) \begin{bmatrix} 1 \\ 0.3846153 \end{bmatrix}$$

$$k = 2:\ A^{-1}x_2 = \begin{bmatrix} 5/14 & 1/7 \\ 3/14 & 2/7 \end{bmatrix} \begin{bmatrix} 1 \\ 5/13 \end{bmatrix} = \begin{bmatrix} 75/182 \\ 59/182 \end{bmatrix}$$

$$= (75/182) \begin{bmatrix} 1 \\ 59/75 \end{bmatrix} = (75/182) \begin{bmatrix} 1 \\ 0.7866666 \end{bmatrix}$$

$$k = 3:\ A^{-1}x_3 = \begin{bmatrix} 5/14 & 1/7 \\ 3/14 & 2/7 \end{bmatrix} \begin{bmatrix} 1 \\ 59/75 \end{bmatrix} = \begin{bmatrix} 493/1050 \\ 461/1050 \end{bmatrix}$$

$$= (493/1050) \begin{bmatrix} 1 \\ 461/493 \end{bmatrix} = (493/1050) \begin{bmatrix} 1 \\ 0.9350912 \end{bmatrix}$$

9.3(c) Shifted Power Method: หา $(A - sI)^{-1}$, $s = 3$

$$(A-3I) = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$$

$$(A - 3I)^{-1} = \begin{bmatrix} -1/2 & -1/2 \\ -3/4 & -1/4 \end{bmatrix}$$

$$k = 0: (A - 3I)^{-1}x_0 = \begin{bmatrix} -1/2 & -1/2 \\ -3/4 & -1/4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1/2 \end{bmatrix}$$

$$= (-1/2) \begin{bmatrix} 0 \\ 1 \end{bmatrix} ,$$

$$k = 1: (A - 3I)^{-1}x_1 = \begin{bmatrix} -1/2 & -1/2 \\ -3/4 & -1/4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1/4 \end{bmatrix}$$

$$= (-1/2) \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$$

$$k = 2: (A - 3I)^{-1}x_2 = \begin{bmatrix} -1/2 & -1/2 \\ -3/4 & -1/4 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} = \begin{bmatrix} -6/8 \\ -7/8 \end{bmatrix}$$

$$= (-7/8) \begin{bmatrix} 6/7 \\ 1 \end{bmatrix} = (-7/8) \begin{bmatrix} 0.8571428 \\ 1 \end{bmatrix}$$

$$k = 3: \quad (A - 3I)^{-1} x_3 = \begin{bmatrix} -1/2 & -1/2 \\ -3/4 & -1/4 \end{bmatrix} \begin{bmatrix} 6/7 \\ 1 \end{bmatrix} = \begin{bmatrix} -26/28 \\ -25/28 \end{bmatrix}$$

$$= (-26/28) \begin{bmatrix} 1 \\ 25/26 \end{bmatrix} = (-26/28) \begin{bmatrix} 1 \\ 0.9615384 \end{bmatrix}$$

9.4

$$A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad s = 3$$

Eigenpairs: $\left[-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right], \left[7, \begin{bmatrix} 5 \\ 3 \end{bmatrix} \right]$

9.4(a) Scaled Power Method: Ax_k = Big $X_i \; x_{k+1}$

$$k = 0: \quad Ax_0 = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 13 \\ 8 \end{bmatrix} = 13 \begin{bmatrix} 1 \\ 8/13 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ 0.6153846 \end{bmatrix}$$

$$k = 1: \ Ax_1 = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6153846 \end{bmatrix} = \begin{bmatrix} 7.0769230 \\ 4.2307692 \end{bmatrix}$$

$$= 7.076923 \begin{bmatrix} 1 \\ 0.597826 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 0.597826 \end{bmatrix}$$

$$k = 2: \ Ax_2 = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0.597826 \end{bmatrix} = \begin{bmatrix} 6.98913 \\ 4.195652 \end{bmatrix}$$

$$= 6.98913 \begin{bmatrix} 1 \\ 0.600311 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 \\ 0.600311 \end{bmatrix}$$

$$k = 3: \ Ax_3 = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0.600311 \end{bmatrix} = \begin{bmatrix} 7.001555 \\ 4.200622 \end{bmatrix}$$

$$= 7.001555 \begin{bmatrix} 1 \\ 0.599955 \end{bmatrix}$$

9.4(b) Inverse Power Method

$$A^{-1} = \begin{bmatrix} -2/7 & 5/7 \\ 3/7 & -4/7 \end{bmatrix}$$

$$\mathbf{x_o} = [2 \quad 1]'$$

$$k = 0: \ \mathbf{A^{-1}x_o} = \begin{bmatrix} -2/7 & 5/7 \\ 3/7 & -4/7 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/7 \\ 2/7 \end{bmatrix}$$

$$= (2/7) \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$k = 1: \ \mathbf{A^{-1}x_1} = \begin{bmatrix} -2/7 & 5/7 \\ 3/7 & -4/7 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8/14 \\ -5/14 \end{bmatrix}$$

$$= (8/14) \begin{bmatrix} 1 \\ -5/8 \end{bmatrix} = (8/14) \begin{bmatrix} 1 \\ -0.625 \end{bmatrix}$$

1 1

$$k = 2: \ \mathbf{A^{-1}x_2} = \begin{bmatrix} -2/7 & 5/7 \\ 3/7 & -4/7 \end{bmatrix} \begin{bmatrix} 1 \\ -5/8 \end{bmatrix} = \begin{bmatrix} -41/56 \\ 44/56 \end{bmatrix}$$

$$= (44/56)\begin{bmatrix} -41/44 \\ 1 \end{bmatrix} = (44/56)\begin{bmatrix} -0.9318181 \\ 1 \end{bmatrix}$$

$$k = 3: \; A^{-1}x_3 = \begin{bmatrix} -2/7 & 5/7 \\ 3/7 & -4/7 \end{bmatrix}\begin{bmatrix} 1 \\ -41/44 \end{bmatrix} = \begin{bmatrix} -2991308 \\ 302/308 \end{bmatrix}$$

$$= (302/308)\begin{bmatrix} 1 \\ -299/302 \end{bmatrix} = (302/308)\begin{bmatrix} 1 \\ -0.9900662 \end{bmatrix}$$

9.4(c) Shifted Power Method: หา $(A - sI)^{-1}$, $s = 3$

$$(A - 3I) = \begin{bmatrix} & \\ 3 & \end{bmatrix}$$

$$(A - 3I)^{-1} = \begin{bmatrix} 1/16 & 5/16 \\ 3/16 & -1/16 \end{bmatrix}$$

$$k = 0: \; (A - 3I)^{-1}x_0 = \begin{bmatrix} 3/16 & -1/16 \\ 1/16 & 5/16 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5/16 \\ 27/16 \end{bmatrix}$$

$$= (7/16)\begin{bmatrix} 1 \\ 5/7 \end{bmatrix} = (7/16)\begin{bmatrix} 1 \\ 0.7142857 \end{bmatrix}$$

$$k = 1: \ (A - 3I)^{-1}x_1 = \begin{bmatrix} 3/16 & -1116 \\ 1/16 & 5/16 \end{bmatrix} \begin{bmatrix} 1 \\ 5/7 \end{bmatrix} = \begin{bmatrix} 2/7 \\ 1/7 \end{bmatrix}$$

$$= (2/7) \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$k = 2: \ (A - 3I)^{-1}x_2 = \begin{bmatrix} 1/16 & 5/16 \\ 3/16 & -1/16 \end{bmatrix} \begin{bmatrix} 1 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 7/32 \\ 5/32 \end{bmatrix}$$

$$= (7/32) \begin{bmatrix} 1 \\ 5/7 \end{bmatrix} = (7/32) \begin{bmatrix} 1 \\ 0.7142857 \end{bmatrix}$$

$$k = 3: \ (A - 3I)^{-1}x_3 = \begin{bmatrix} 1/16 & 5/16 \\ 3/16 & -1/16 \end{bmatrix} \begin{bmatrix} 1 \\ 5/7 \end{bmatrix} = \begin{bmatrix} 2/7 \\ 1/7 \end{bmatrix}$$

$$= (2/7) \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$$

9.5 จงแสดงว่า $(AB)' = B'A'$ เมื่อ

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ และ } B = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix}$$

$$AR = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 21 & 24 & 7 \\ 47 & 54 & 21 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 21 & 47 \\ 24 & 54 \\ 7 & 21 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 5 & 8 \\ 6 & 9 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 21 & 47 \\ 24 & 54 \\ 7 & 21 \end{bmatrix}$$

$$(AB)' = B'A'$$

9.6 สำหรับ $x = [1 \ -1 \quad]'$ และ $\mathbf{y} = [2 \ \ 3 \ \ -1]'$ จงหา $\cos \underline{/}(\mathbf{x},\mathbf{y})$

$$\mathbf{x'y} = [1 \ \ -1 \ \ 1] \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = 2 - 3 - 1 = -2$$

$$\left\|\, \mathbf{x} \,\right\| = (3)^{(1/2)} = (\mathbf{x'x})^{(1/2)}$$

$$\left\|\, \mathbf{y} \,\right\| = (14)^{(1/2)} = (\mathbf{y'y})^{(1/2)}$$

$$\cos\underline{/}(\mathbf{x},\mathbf{y}) = \mathbf{x'y} \,/\, (\left\|\, \mathbf{x} \,\right\| \left\|\, \mathbf{y} \,\right\|)$$

$$= (-2)/[(3)^{(1/2)}(14)^{(1/2)}]$$

$$= -2/(42)^{(1/2)}$$

9.7 ใน (a)-(b) จงใช้วิธีของยาโคบิ เพื่อหา rotation matrix U และ diagonal matrix $D = diag(\lambda_1, \lambda_2)$ ซึ่ง $U'AU = D$

(a) $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $B = \begin{bmatrix} -23 & 36 \\ 36 & -2 \end{bmatrix}$

9.7(a) จากการเรียกใช้ซับรูทีน ROTATE ในภาคผนวก ค (บทที่ 9) ได้ผลดังนี้

```
N=    2

A(I,J)  BY COLUMN:    2.0000000    1.0000000    1.0000000
                      2.0000000

NUMDEC  =   7  ,MAXIT  =  10

ABSTOL  =      .0000001

 K II JJ      A(II,JJ)      AMAXOD

 1  1  2    1.0000000     1.0000000

A(I,J)  BY  COLUMN:   3.0000000     .0000000     .0000000
                       .9999999

U(I,J)  BY  COLUMN:    .7071068     .7071068    -.7071068
                       .7071068

 K  II  JJ      A(II,JJ)       AMAXOD

 2   12       .0000000      .0000000

A(I,J)  BY  COLUMN:   3.0000000     .0000000     .0000000
                       .9999999

U(I,J)  BY COLUMN:     .7071068     .7071068    -.7071068
                       .7071068

NO.  OF  ROTATIONS  PERFORMED  =  2

EIGEN  VALUE    3.0000000
EIGEN   VECTOR:  .7071068     .7071068

EIGEN  VALUE     .9999999
EIGEN  VECTOR:   -.7071068     .7071068
```

3.7(b) จากการเรียกใช้ซับรูทีน ROTATE ในภาคผนวก ค (บทที่ 9) ได้ผลดังนี้

```
N=   2

A(I,J) BY COLUMN: -23.0000000  36.0000000  36.0000000
                   -2.0000000

NUMDEC  =   7 ,MAXIT = 10

ABSTOL  =     .0000001

 K II JJ      A(II,JJ)     AMAXOD

 1  1  2  36.0000000  36.0000000

A(I,J) BY COLUMN:  -50.0000000     . 0000000     .0000000
                    25.0000000

U(I,J) BY COLUMN:     .8000000   -.6000000    .6000000
                      .8000000

 K II JJ      A(II,JJ)     AMAXOD

 2  1  2     .0000000     .0000000

A(I,J)  BY  COLUMN:  -50.0000000    , 0000000     .0000000
                     25.0000000

U(I ,J) BY COLUMN:    .8000000   -.6000000    .6000000
                      .8000000

NO.  OF  ROTATIONS  PERFORMED  =  2

EIGEN  VALUE  -50.0000000
EIGEN  VECTOR:     .8000000   -.6000000

EIGEN   VALUE   25.0000000
EIGEN  VECTOR:     .6000000    .8000000
```

9.8 สำหรับเมตริกซ์ $A = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix}$ จงทำ 2 iterations ของ

วิธีของยาโคบิ

จากการเรียกใช้ซับรูทีน ROTATE ในภาคผนวก ค (บทที่ 9) ได้ผลดังนี้

```
N =   3

A(I,J)  BY COLUMN:     1.0000000   -1.0000000     2.0000000
                      -1.0000000    1.0000000      .0000000

                       2.0000000     .0000000     2.0000000

NUMDEC  =   7  ,MAXIT  =  10

ABSTOL  =      .0000001

 K II JJ     A(II,JJ)      AMAXOD

 1  1  3    2.0000000    2.0000000

A(I,J)  BY COLUMN:     -.5615529   -.7882054      .0000000
                       -.7882054   1.0000000    -.6154122

                        .0000000   -.6154122    3.5615530

U(I,J)  BY COLUMN:      .7882054    .0000000    -.6154122
                        .0000000   1.0000000      .0000000

                        .6154122    .0000000     .7882054
```

```
K II JJ     A(II,JJ)      AMAXOD

2  1  2    -.7882054     .7882054

A(I,J) BY COLUMN:     -.8902266     .0000000    -.2368535
                       .0000000    1.3286740    -.5680076

                      -.2368535    -.5680076    3.5615530

U(I,J) BY COLUMN:      .7274907     .3848696    -.5680076
                      -.3033563     .9229710     .2368535

                       .6154122     .0000000     .7882054

K II JJ     A(II,JJ)      AMAXQD

3  2  3    -.5680076     .5680076

A(I,J) BY COLUMN: -. 8902266       -.0552230    -.2303258
                      -.0552230    1.1924880     .0000000

                      -.2303258     .0000000    3.6977390

U(I,J) BY COLUMN:      .7274907     .3848696    -.5680076
                      -.1515110     .a975340    , 4140979

                       .6691799    -.2151930     .7112597

K II JJ     A(II,JJ)      AMAXOD

4  1  3    -.2303258     .2303258

A(I,J) RY COLUMN:     -.9017605    -.0551539     0000000
                      -.0551539    1.1924880     .0027619

                     ,  0000000     .0027619    3.7092720

U(I,J) BY COLUMN:      .7600484     .3736254    -.5317241
                      -.I515110     .a975340     .4140979

                       .6319579    -.2341724     .7387776

K II JJ     A(II,JJ)      AMAXOD

5  1  2    -.0551539     .0551539

A(I,J) BY COLUMN>     -.9032120     .0000000     .0000727
                       .0000000    1.1939400     .0027609

                       .0000727     .0027609    3.7092720

U(I,J) BY COLUMN:      .7557993     .3971089    -.5206457
                      -.1714543     .8873938     .4279434

                       .6319579    -.2341724     .7387776
```

```
 K  II  JJ      A(II,JJ)       AMAXOD

 6   2   3     .0027609     .0027609

A(I,J)  BY  COLUMN:    -.9032120    -.0000001     .0000727
                       -.0000001    1.1939370     .0000000

                        .0000727    . 0000000    3.7092750

U(I,J)  BY  COLUMN:     .7557993     .3971089    -.5206457
                       -.1721479     .8876504     .4271323

                        .6317694    -.2331982     .7392469

 K  II  JJ      A(II,JJ)       AMAXOD

 7   1   3     .0000727     .0000727

A(I,J)  BY  COLUMN:    -.9032120    -.0000001     .0000000
                       -.0000001    1.1939370     .0000000

                        .0000000    . 0000000    3.7092750

U(I,J)  BY  COLUMN:     .7557893     .3971125    -.5206574
                       -. 1721479    .8876504     .4271323

                        .6317813    -.2331920     .7392387

 K  II  JJ      A(II,JJ))      AMAXOD

 8   1   2    -.0000001     .0000001

A(I,J)  BY  COLUMN:    -.9032120    . 0000000     .0000000
                        .0000000    1.1939370    . 0000000

                        .0000000    . 0000000    3.7092750

U(I,J)  BY  COLUMN:     .7557893     .3971125    -.5206574
                       -. 1721479    .8876504     .4271323

                        .6317813    -.2331920     .7392387

NO.  OF  ROTATIONS  PERFORMED  =  8

EIGEN  VALUE   -.9032120
EIGEN  VECTOR:     .7557893     .3971125    -.5206574

EIGEN  VALUE    1.1939370
EIGEN  VECTOR:    -.1721479     .8876504     .4271323

EIGEN  VALUE    3.7092750
EIGEN  VECTOR:     .6317813    -.2331920     .7392387
```