เฉลยแบบฝึกหัดบทที่ 6

6.1 For the cubic knots $P_1(0,0)$, $P_2(1,1)$, $P_3(2,8)$, $P_4(3,27)$, find $p_{1,3}(x)$ two ways:

(a) Use the method of undetermined coefficients (Section 6.1A).

(b) Use the Lagrange form of $p_{1,3}(x)$ (Section 6.1C)

Ans. $p_{1,3}(x) = 6x^2 - 11x + 6$

(a) $p_{1,3}(x) = y = A + Bx + Cx^2$

$x = 1,\ p_{1,3}(1) = y_2 = A + B + C = 1 \qquad \ldots(1)$

$x = 2,\ p_{1,3}(2) = y_3 = A + 2B + 4C = 8 \qquad \ldots(2)$

$x = 3,\ p_{1,3}(3) = y_4 = A + 3B + 9C = 27 \qquad \ldots(3)$

(2)-(1); $\qquad B + 3C = 7 \qquad \ldots(4)$

(3)-(2); $\qquad B + 5C = 19 \qquad \ldots(5)$

(5)-(4); $\qquad 2C = 12 \qquad \ldots(6)$

$\qquad C = 6$

จาก (5); $\qquad B = -11$

จาก (1); $\qquad A = 6$

ดังนั้น $\qquad p_{1,3}(x) = 6x^2 - 11x + 6$

(b) $p_{1,3}(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$

$$= y_1\frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2\frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + y_3\frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$= 1\,\frac{(x-2)(x-3)}{(1-2)(1-3)} + 8\,\frac{(x-1)(x-3)}{(2-1)(2-3)} + 27\,\frac{(x-1)(x-2)}{(3-1)(3-2)}$$

$$= \frac{(x^2-5x+6)}{2} + \frac{8(x^2-4x+3)}{-1} + \frac{27(x^2-3x+2)}{}$$

$$= (1/2)(12x^2 - 22x + 12)$$

$$= 6x^2 - 11x + 6$$

6.2 For the knots $P_0(-2,-15), P_1(-1,-2), P_2(0,1), P_3(2,1), P_4(3,10)$ find the Lagrange form (do not simplify) of

(a) $p_{2,3}(x)$ (b) $p_{2,4}(x)$ (c) $p_{1,4}(x)$ (d) $p_{0,4}(x)$

(a) $p_{2,3}(x) = y_2 L_2(x) + y_3 L_3(x)$

$$= y_2 \frac{(x-x_3)}{(x_2-x_1)} + y_3 \frac{(x-x_2)}{(X,-X,)}$$

$$= 1 \frac{(x-2)}{(0-2)} + 1 \frac{(x-0)}{(2-0)}$$

$$= (-1/2)(x-2) + (1/2)x = 1$$

(b) $p_{2,4}(x) = y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x)$

$$= y_2 \frac{(x-x_3)(x-x_4)}{(x_2-x_3)(x_2-x_4)} + y_3 \frac{(x-x_2)(x-x_4)}{(x_3-x_2)(x_3-x_4)} + y_4 \frac{(x-x_2)(x-x_3)}{(x,-x,)(x,-x,)}$$

$$1 \frac{(x-2)(x-3)}{(0-2)(0-3)} + 1 \frac{(x-0)(x-3)}{(2-0)(2-3)} + 10 \frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$$= (1/6)(x-2)(x-3) - (1/2)x(x-3)\ t\ (10/3)x(x-2)$$

(c) $p_{1,4}(x) = y_1 L_1(x)\ t\ y_2 L_2(x)\ t\ y_3 L_3(x)\ t\ y_4 L_4(x)$

$$= y_1 \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + y_2 \frac{(X-X,)(X-X,)(X-X,)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$+ y_3 \frac{(X-X,)(X-X,)(X-X,)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} + y_4 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$= -2 \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)} + 1 \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}$$

$$t\ 1 \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)}\ t\ 10 \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}$$

$$= (1/6)x(x-2)(x-3)\ t\ (1/6)(x+1)(x-2)(x-3)$$

$$- (1/6)(x+1)x(x-3)\ t\ (5/6)x(x+1)(x-2)$$

(d) $p_{0,4}(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)\ t\ y_3 L_3(x)\ t\ y_4 L(x_4)$

$$= y_0 \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$+ y_1 \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$+ y_2 \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$+ y_3 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$+ y_4 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$=(-15)\frac{(x-0)(x+1)(x-2)(x-3)}{(-2-0)(-2+1)(-2-2)(-2-3)}+(-2)\frac{(x+2)(x-0)(x-2)(x-3)}{(-1+2)(-1-0)(-1-2)(-1-3)}$$

$$+ 1 \frac{(x+2)(x+1)(x-2)(x-3)}{(0+2)(0+1)(0-2)(0-3)} + 1 \frac{(x+2)(x+1)(x-0)(x-3)}{(2+2)(2+1)(2-0)(2-3)}$$

$$+ 10 \frac{(x+2)(x+1)(x-0)(x-2)}{(3+2)(3+1)(3-0)(3-2)}$$

$$= (-3/8)x(x+1)(x-2)(x-3) \text{ t } (1/6)x(x+2)(x-2)(x-3)$$

$$(1/12)(x+2)(x+1)(x-2)(x-3) - (1/24)x(x+2)(x+1)(x-3)$$

$$(1/6)x(x+2)(x+1)(x-2)$$

6.3 For the knots $P_0(-3,1), P_1(0,9), P_2(2,1), P_3(3,1), P_4(5,81)$, find the Lagrange form (do not, simplify) of

(a) $p_{1,2}(x)$ (b) $p_{0,2}(x)$ (c) $p_{0,3}(x)$ (d) $p_{0,4}(x)$

(a) $p_{1,2}(x) = y_1 L_1(x) \text{ t } y_2 L_2(x)$

$$= y_1 \frac{(x-x_2)}{(x_1-x_2)} + y_2 \frac{(x-x_1)}{(x_2-x_1)}$$

$$3 \frac{(X-2)}{(0-2)} \text{ t } 1 \frac{(X-0)'}{(2-0)}$$

(b) $p_{0,2}(x) = y_0 L_0(x) \text{ t } y_1 L_1(x) \text{ t } y_2 L_2(x)$

$$= y_0 \frac{(x-x_1)(x-x_2)}{(X,-X,)(X,-X,)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= 1 \frac{(x-0)\ (x-21}{(-3-0)(-3-2)} \text{ t } 3 \frac{(x+3)\ (x-2)}{(0+3)(0-2)} \text{ t } 1 \frac{(x+3)(x-0)}{(2+3)(2-0)}$$

(c) $p_{0,3}(x) = y_0 L_0(x) \text{ t } y_1 L_1(x) \text{ t } y_2 L_2(x) \text{ t } y_3 L_3(x)$

$$= y_0 \frac{(X-X,)(X-X,)(X-X,)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)\,(X-X,)\,(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ y_2 \frac{(X-X,)(X-X,)(X-X,)}{(X,-X,)(X,-X,)(X,-X,)} \text{ t } v_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$1 \frac{(x-0)\,(x-2)\,(x-3)}{(-3-0)(-3-2)(-3-3)} \text{ t } 9 \frac{(x+3)(x-2)(x-3)}{(0+3)(0-2)(0-3)}$$

$$\text{t } 1 \frac{(x+3)(x-0)(x-3)}{(2+3)(2-0)(2-3)} \text{ t } 1 \frac{(x+3)(x-0)(x-2)}{(3+3)(3-0)(3-2)}$$

(d) $p_{0,4}(x) = y_0 L_0(x) \text{ t } y_1 L_1(x) + y_2 L_2(x) \text{ t } y_3 L_3(x) \text{ t } y_4 L(x_4)$

$$= y_0 \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$+ y_1 \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$+\; y_2 \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$+\; y_3 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}$$

$$+\; y_4 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$= 1 \frac{(x-0)(x-2)(x-3)(x-5)}{(-3-0)(-3-2)(-3-3)(-3-5)} + 9 \frac{(x+3)(x-2)(x-3)(x-5)}{(0+3)(0-2)(0-3)(0-5)}$$

$$+\; 1 \frac{(x+3)(x-0)(x-3)(x-5)}{(2+3)(2-0)(2-3)(2-5)} + 1 \frac{(x+3)(x-0)(x-2)(x-5)}{(3+3)(3-0)(3-2)(3-5)}$$

$$+\; 81 \frac{(x+3)(x-0)(x-2)(x-3)}{(5+3)(5-0)(5-2)(5-3)}$$

6.4 Let $L_0(x), \ldots, L_4(x)$ denote the Lagrange polynomials for $P_0, \ldots, P_4$.

(a) For P,,...,P_4 of Exercise 6.2, use the selecting property (6) of Section 6.1C to evaluate *by inspect ion*

(i) $L_2(3)$ (ii) $L_2(0)$ (iii) $L_3(3)$ (iv) $L_3(2)$ (v) $L_2(2)$

(b) Repeat part (a) for $P_0,\ldots,P_4$ of Exercise 6.3.

(a) (i) $L_2(3) = L_2(x_4) = 0$

(ii) $L_2(0) = L_2(x_2) = 1$

(iii) $L_3(3) = L_3(x_4) = 0$

(iv) $L_3(2) = L_3(x_3) = 1$

(v) $L_2(2) = L_2(x_3) = 0$

(b) (i) $L_2(3) = L_2(x_3) = 0$

(ii) $L_2(0) = L_2(x_1) = 0$

(iii) $L_3(3) = L_3(x_3) = 1$

(iv) $L_3(2) = L_3(x_2) = 0$

(v) $L_2(2) = L_2(x_2) = 1$

6.5 Let $x_0,x_1,\ldots,x_n$ denote any ntl distinct nodes. Use the uniqueness of $p_{0,n}(x)$ to show that their Lagrange polyniaials $L_0(x),\ldots,L_n(x)$ must satisfy

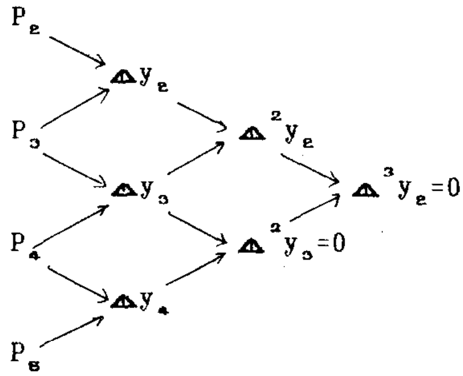
(a) $L_0(x)+L_1(x)+\ldots+L_n(x) = 1$(constant) for all x.

(b) $x_0L_0(x)+x_1L_1(x)+\ldots+x_nL_n(x) = x$ for all x.

You do not have to write out the $L_j(x)$'s. Consider $f(x)=1$ in part (a) and $f(x)=x$ in part (b).

ให้นักศึกษาทำเอง

6.6 What (if anything) can you conclude about the knots P_2, P_3, P_4, P_5 if $\Delta^3 y_2 = 0$? If $\Delta^2 y_3 = 0$?



ถ้า $\Delta^3 y_2 = 0$, $p_{2,5}(x)$ เป็น polynomial function ซึ่งมี degree $<= 2$

ถ้า $\Delta^2 y_3 = 0$, $p_{3,5}(x)$ เป็นสมการเส้นตรง (polynomial function ซึ่งมี degree $<= 1$)

6.7 Consider the knots $P_0(-3,1)$, $P_1(-1,9)$, $P_2(0,1)$, $P_3(3,1)$, $P_4(5,-39)$.

(a) Form a DD **table** for $P_0,\ldots,P_4$ and use it in (b)-(d).

(b) What are the values of $\Delta^1 y_3$, $\Delta^3 y_1$, $\Delta^4 y_0$, $\Delta^2 y_1$, and $\Delta^2 y_2$?

(c) Find (in the order given): $p_{1,2}(x)$, $p_{1,3}(x)$, $p_{0,3}(x)$, P,*,(X)-

(d) Find (in the order given): $p_{2,3}(x)$, $p_{1,3}(x)$, $p_{1,4}(x)$, $p_{0,4}(x)$.

(a)

Knot	Node	y				
P_0	-3	1	Δ			
			4	Δ^2		
P_1	-1	9		-4	Δ^3	
			-0		1	Δ^4
P_2	0	1		2		-1/4
			0		-1	
P_3	3	1		-4		
			-20			
P_4	5	-39				

(b) $\Delta^1 y_3 = -20$, $\Delta^3 y_1 = -1$, $\Delta^4 y_0 = -1/4$, $\Delta^2 y_1 = 2$, $\Delta^2 y_2 = -4$

(c) $p_{1,2}(x) = 9 + (-8)(x+1) = 1 - 8x$

$p_{1,3}(x) = 9 + (-8)(x+1) + 2(x+1)(x-0)$

$= 2x^2 - 6x + 1$

$p_{0,3}(x) = 9 + (-8)(x+1) + 2(x+1)(x-0) + 1(x+1)(x-0)(x-3)$

$= x^3 - 9x + 1$

$p_{0,4}(x) = 9 + (-8)(x+1) + 2(x+1)(x-0) + 1(x+1)(x-0)(x-3)$

$- (1/4)(x+1)(x-0)(x-3)(x+3)$

$= (-1/4)x^4 + (3/4)x^3 + (9/4)x^2 - (27/4)x +$

(d) $p_{2,3}(x) = 1 - o(X-0) = 1$

$p_{1,3}(x) = 1 - 0(x-0) + 2x(x-3) = 2x^2 - 6x + 1$

$p_{1,4}(x) = 1 - 0(x-0) + 2x(x-3) + (-1)x(x-3)(x+1)$

$= -x^3 + 4x^2 - 3x + 1$

$p_{0,4}(x) = 1 - 0(x-0) + 2x(x-3) + (-1)x(x-3)(x+1)$

$- (1/4)x(x-3)(x+1)(x-5)$

$= (-1/4)x^4 + (3/4)x^3 + (9/4)x^2 - (27/4)x + 1$

6.8 Complete the following DD table up to the Δ^3 column.

P_k	x_k	y_k	Δ^1	Δ^2	Δ^3
P_0	-3				
P_1	-2				
				17	13
P_2	-1		1		
P_3	0		49	67	
P_4	3				
P_5	4	463			

P_k	x_k	y_k	Δ^1	Δ^2	Δ^3
P_0	-3	20			
			11		
P_1	-2	31		-22	
			-33		13
P_2	-1	-2		17	
			1		-1
P_3	0	-1		12	
			49		11
P_4	3	146		67	
			317		
P_5	4	463			

6.9 For the DD table shown, form [without simplifying, as in (17) in section 6.2D] $p_{0,5}(x)$ by adding nodes in the following orders:

(a) $x_0, x_1, x_2, x_3, x_4, x_5$ [forward: use only (12a) of Section 6.2B]

(b) $x_5, x_4, x_3, x_2, x_1, x_0$ [backward: use only (12b) of Section 6.2B]

(c) $x_2, x_1, x_3, x_4, x_5, x_0$

(d) $x_3, x_2, x_1, x_4, x_0, x_5$

Circle the leading coefficients used.

Knot	Node	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
P_0	-2	21					
			-14				
P_1	-1	7		4			
			-6		-1		
P_2	0	1		1		0	
			-4		-1		1
P_3	1	-3		-2		5	
			-8		19		
P_4	2	-11		55			
			102				
P_5	3	91					

(a) $p_{0,5}(x) = 21 - 14(x+2) + 4(x+2)(x+1) - 1(x+2)(x+1)(x-0)$
$+ 0(x+2)(x+1)(x-0)(x-1) + 1(x+2)(x+1)(x-0)(x-1)(x-2)$

(b) $p_{0,5}(x) = 91 + 102(x-3) + 55(x-3)(x-2) + 19(x-3)(x-2)(x-1)$
$+ 5(x-3)(x-2)(x-1)(x-0) + 1(x-3)(x-2)(x-1)(x-0)(x+1)$

(c) $p_{0,5}(x) = 1 \ -6(x-0) +1(x-0)(x+1) -1(x-0)(x+1)(x-1)$
$+5(x-0)(x+1)(x-1)(x-2) +1(x-0)(x+1)(x-1)(x-2)(x-3)$

(d) $p_{0,5}(x) = -3 \ -4(x-1) +1(x-1)x -1(x-1)x(x+1)$
$+0(x-1)x(x+1)(x-2) +1(x-1)x(x+1)(x-2)(x+2)$

6.10 For (a)-(d) of Exercise 6.9, form $p_{0,5}(x)$ in nested form, and use it to evaluate $p_{0,5}(-3)$.

(a) $[[[[1(x-2)(x-1)-1]x+4](x+1)-14](x+2)+21$

(b) $[[[[1(x+1)+5](x-0)+19](x-1)+55](x-2)+102](x-3)+91$

(c) $[[[[1(x-3)+5](x-2)-1](x-1)+1](x+1)-6](x-0)+1$

(d) $[[[[1(x+2)+0](x-2)-1](x+1)+1](x-0)-4](x-1)-3$

จาก (a)-(d) $p_{0,5}(-3) = -71$

6.11 By inspection of the DD table in Exercise 6.9, find the leading coefficient. of $p_{1,3}(x)$, $p_{2,5}(x)$, and $p_{0,4}(x)$.

leading coefficient. of $p_{1,3}(x) = 1$
leading coefficient of $p_{2,5}(x) = 19$
leading coefficient of $p_{0,4}(x) = 0$

6.12 Form a DD table for the six knots
$P_0(-3,200), P_1(-2,46), P_2(-1,6), P_3(0,2), P_4(2,30), P_5(3,146)$
and use it to determine the degree of $p_{0,5}(x)$.

Knot	Node	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
P_0	-3	200					
			154				
P_1	-2	46		57			
			-40		-13		
P_2	-1	6		18		2	
			-4		-3		0
P_3	0	2		6		2	
			14		7		
P_4	2	30		34			
			116				
P_5	3	146					

Degree ของ $p_{0,5}(x) = 4$

6.13 For $P_0, \ldots, P_5$ of Exercise 6.12, find $p_{0,5}(x)$ as indicated in (a)-(d) of Exercise 6.9. Use any nested form to evaluate $p_{0,5}(1)$.

ให้นักศึกษาทำเอง และ $p_{0,5}(1) = 4$

6.14 Make a forward difference table for the knots used in Exercise 6.9. Verify that $\Delta^m y_k = h^m m! \Delta^m y_k$ for $\Delta^3 y_0$, $\Delta^4 y_1$, and $\Delta^2 y_2$.

Forward difference table

Knot	Node	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
P_0	-2	21					
			-14				
P_1	-1	7		8			
			-6		-6		
P_2	0	1		0 2		0	
			-4		-6		0 120
P_3	1	-3		-4		120	
			-8		114		
P_4	2	-11		110			
			102				
P_5	3	91					

ตรวจสอบว่า $\Delta^m y_k = h^m m! \Delta^m y_k$

$$\Delta^3 y_0 = (1)(3!)\Delta^3 y_0 = 6(-1) = -6$$

$$\Delta^4 y_1 = (1)(4!)\Delta^4 y_1 = 24(5) = 120$$

$$\Delta^2 y_2 = (1)(2!)\Delta^2 y_2 = 2(1) = 2$$

6.15 (a) Show that the entries of a forward difference table with h-spaced nodes can be used to get the forward recursive formula

$$p_{k,k+m}(x) = p_{k,k+m-1}(x) + (\Delta^m y_k / m! h^m)(x-x_k)(x-x_{k+1})\ldots(x-x_{k+m-1})$$

(b) **Deduce the Newton foruard difference formula** for $m >= 1$:

$$P_{k,k+m}(x) = y_k + (\Delta y_k/(1!\ h))(x-x_k)+(\Delta^2 y_k/(2!\ h^2))(x-x_k)(x-x_k-h)$$

$$+\ldots+(\Delta^m y_k/(m!\ h^m)) \prod_{j=0}^{m-1} (x-x_k-jh)$$

(c) Conjecture a ***backward*** difference formula for $P_{k,k+m}(x)$.

6.16 Make a forward difference table for the following data: $P_0(1,263), P_1(1.5,230), P_2(2,200), P_3(2.5,174), P_4(3,150)$ Use it to find the Newton forward difference formula for $p_{0,4}(x)$.

Forward difference table

Knot	Node	y	Δ	Δ^2	Δ^3	Δ^4
P_0	1	263				
			-33			
P_1	1.5	230		3		
			-30		1	
P_2	2	200		4		-3
			-26		-2	
P_3	2.5	174		2		
			-24			
P_4	3	150				

จาก 6.15(b) $k = 0$, $m = 4$, $h = 0.5$

$$P_{0,4}(x) = y_0 + [\Delta y_0/(1!\ (0.5))](x-x_0)$$
$$+ [\Delta^2 y_0/(2!\ (0.5)^2)](x-x_0)(x-x_0-0.5)$$
$$+ [\Delta^3 y_0/(3!\ (0.5)^3)](x-x_0)(x-x_0-0.5)(x-x_0-1)$$
$$+ [\Delta^4 y_0/(4!\ (0.5)^4)](x-x_0)(x-x_0-0.5)(x-x_0-1)(x-x_0-1.5)$$

$$= 263 + [(-33)/0.5](x-1) + [3/2(.25)](x-1)(x-1.5)$$
$$+ [1/6(0.5)^3](x-1)(x-1.5)(x-2)$$
$$+ [(-3)/24(0.5)^4](x-1)(x-1.5)(x-2)(x-2.5)$$

$$= 263 - 66(x-1) + 6(x-1)(x-1.5)$$
$$+ (4/3)(x-1)(x-1.5)(x-2) - 2(x-1)(x-1.5)(x-2)(x-2.5)$$

6.17 Using the 5s DD Table of Figure 6.3-2, set up a Worksheet Table as in 6.3-2 for getting $\hat{p}_0(z),\ldots,\hat{p}_5(z)$ when z is (a) 0.1 (b) 0.75

[Note: When z is the midpoint of $[x_k, x_{k+1}]$, take either y_k or y_{k+1} as $\hat{p}_0(z)$.]

(a) $z = 0.1$, $\hat{p}_0(z) = 0.5793$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.2		0.5793
1	0.0	-0.03965	0.53965
2	0.4	0.0004	0.54005
3	0.6	-0.0001625	0.5396675
4	0.8	-0.0000196	0.5398679
5	1.0	0.0000081	0.5398761

(b) $z = 0.75$, $\hat{p}_0(z) = 0.7881$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.8		0.7881
1	0.6	-0.0156	0.7725
2	1.0	0.0008625	0.7733625
3	0.4	-0.0000508	0.7733117
4	0.2	-0.0000137	0.7733254
5	0.0	0.0000027	0.7733281

6.18 Using the 5s DD Table of Figure 6.3-2, estimate $\Phi(-0.5) \doteq$ **0.3085** two ways:

(a) Extrapolate: Find $\hat{p}_m(-0.5)$, $m=0,1,\ldots,5$.

(b) Use the fact that $\Phi(-z)$, $= 1- \Phi(z)$.

Which of parts (a) and (b) is more accurate? Did you expect this? Was the most accurate $\hat{p}_m(z)$ in part (a) the one with for which $\delta_m(z)$ is smallest7 Can you suggest a rule of thumb for which $\hat{p}_m(z)$ to use when $\delta_m(z)$ stops shrinking substantially as m is increased7

(a) $z = -0.5$, $\hat{p}_0(z) = \Phi(0.0) = 0.5$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.0		0.5
1	0.2	-0.19825	0.30175
2	0.4	-0.01400	0.28775
3	0.6	0.0170625	0.3048125
4	0.8	0.0045116	0.3093241 ---> $\hat{p}_4(-0.5) \doteq$ **0.30932**
5	1.0	-0.0035187	0.3058054

$\Phi(-0.5) \approx \hat{p}_4(-0.5) \doteq 0.30932$

(b) $z = 0.5$, $\hat{p}_o(z) = \Phi(0.6) = 0.7257$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$	$1 - \hat{p}_m(z)$
0	0.6		0.7257	0.2743
1	0.4	-0.03515	0.69055	0.30945
2	0.8	0.0009875	0.6915375	0.3084625
3	0.2	-0.0001312	0.6914063	0.3085937
4	1.0	0.0000187	0.6914250	0.3085750
5	0.0	-0.0000035	0.6914215	0.3085785

ดังนั้น $\Phi(-0.5) = 1 - \Phi(0.5) \approx 1 - \hat{p}_5(0.5) \doteq 0.3085785$

6.19 Estimate (a) $\Phi(0.52)$, (b) $\Phi(0.22)$, and (c) $\Phi(1.4)$ using the DD table in Figure 6.3-4; compare your accuracy with that obtained in Table 6.3-2.

(a) $z = 0.52$, $\hat{p}_o(z) = \Phi(0.6) = 0.726$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.6		0.726
1	0.4	-0.0284	0.6976
2	0.8	0.00108	0.69868
3	0.2	-0.0002239	0.6984561
4	1.0	0.0000894	0.6985455
5	0.0	-0.0000645	0.698481

$\Phi(0.52) \approx \hat{p}_5(0.52) \doteq 0.69848$

(b) $z = 0.22$, $\hat{p}_0(z) = \Phi(0.2) = 0.579$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.2		0.579
1	0.4	0.0076	0.5666
2	0.0	0.000135	0.586735
3	0.6	0.000033	0.586768
4	0.8	-0.0000157	0.5867523

$\Phi(0.22) \approx \hat{p}_4(0.22) \doteq 0.58675$

(c) $z = 1.4$, $\hat{p}_0(z) = \Phi(1.0) = 0.841$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	1.0		0.841
1	0.8	0.106	0.947
2	0.6	-0.027	0.9200
3	0.4	0.000	0.9200

$\Phi(1.4) \approx \hat{p}_3(1.4) \doteq 0.9200$

6.20 In (a)-(c), use a 7d DD table based on the 6d values of sinh $x = (1/2)[\exp(x)-\exp(-x)]$ shown in the accompanying table.

x	sinh x
0.2	0.201336
0.4	0.410752
0.6	0.636654
0.8	**0.888106**
1.0	1.175201

(a) With $z = 0.55$, find the best interpolants $\hat{p}_0(z)$, $\hat{p}_1(z), \hat{p}_2(z), \ldots,$ stopping when either $\delta_m(z) = 0$ or $\left|\delta_{m+1}(z)\right| > \left|\delta_m(z)\right|$.

(b) Repeat part (a) with $z =$ **0.66.**

(c) Repeat part (a) with $z = 1.3$ (extrapolation,.

(d) Discuss the accuracy of the approximations $\sinh z \approx \hat{p}_m(z)$ obtained in (a)-(c). Did you expect some to be more accurate than others? Explain.

(a) จากการใช้ซับรูทีน FORMDD ในภาคผนวก ค ได้ผลดังนี้

```
ENTER X'S : .2 .4 .6 .8 1.0

ENTER Y'S : .201336 .410752 .636654 .888106 1.175201


DD( 1, 2) :   1.0470800  .1047080E+01
DD( 2, 2) :   1.1295100  .1129510E+01
DD( 3, 2) :   1.2572600  .1257260E+01
DD( 4, 2) :   1.4354750  .1435475E+01
DD( 1, 3) :    .2060750  .2060750E+00
DD( 2, 3) :    .3193748  .3193748E+00
DD( 3, 3) :    .4455388  .4455388E+00
DD( 1, 4) :    .1888330  .1888330E+00
DD( 2, 4) :    .2102733  .2102733E+00
DD( 1, 5) :    .0268004  .2680039E-01Stop - Program terminated.
```

DD Table คือ

x	y	Δ^1	Δ^2	Δ^3	Δ^4
0.2	0.201336				
		1.04708			
0.4	0.410752		0.206075		
		1.12951		0.1888330	
0.6	0.636654		0.3193748		0.0268004
		1.25726		0.2102733	
0.8	0.888106		0.4455388		
		1.435475			
1.0	1.175201				

(a) $z = 0.55$, $\hat{p}_0(z) = 0.636654$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.6		0.6366540
1	0.4	-0.0564755	0.5801785
2	0.8	-0.0023953	0.5777832
3	0.2	0.000354	0.5781372
4	1.0	0.0000175	0.5781547

(b) $z = 0.86$, $\hat{p}_o(z) = 0.888106$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	0.8		0.888106
1	1.0	0.0861285	0.9742345
2	0.6	-0.0037425	0.970492
3	0.4	-0.0004592	0.9700328
4	0.2	-0.0000269	0.970059

(c) $z = 1.3$, $\hat{p}_o(z) = 1.175201$

m	New node	$\delta_m(z)$	$\hat{p}_m(z)$
0	1.0		1.175201
1	0.8	0.4306425	1.6058435
2	0.6	0.0668308	1.6726743
3	0.4	0.0220786	1.6947529
4	0.2	0.0025326	1.6972855