5.1 Find $\hat{a}$, $\hat{b}$ and $E(\hat{L})$ for $\hat{L}(x) = \hat{a} + \hat{b}x$, the least square straight line for the data shown in (a) and (b).

(a)

x	0	1	2	3
y	3.0	1.2	-0.3	-1.5

(b)

x	1.0	1.2	1.4	1.6	1.8
y	-5	-3	-2	0	3

(a) จากการใช้ซับรูทีน LINFIT ในหัวข้อ 5.1D ได้ผลดังนี้

```
ENTER N X(I),Y(I),I=1,...,N   UNIT 5? CON
4 0 3 1 1.2 2 -.3 3 -1.5

X:      .00000   1.00000   2.00000   3.00000
Y:    3.00000   1.20000   -.30000  -1.50000
ERROR SUM OF SQUARE =     .90000E-01
Y - INTERCEPT         =    .28500E+01
SLOPE                 =   -.15000E+01
PREDICTION EQUATION: Y^=     2.85000 +   -1.50000 *X
```

$\hat{L}(x) = 2.85 - 1.5x$

$E(\hat{L}) = 0.09$

(b) จากการใช้ซับรูทีน LINFIT ในหัวข้อ 5.1D ได้ผลดังนี้

```
ENTER N X(I),Y(I),I=1,...,N   UNIT 5? CON
5 1 -5 1.2 -3 1.4 -2 1.6 0 1.8 3

X:    1.00000   1.20000   1.40000   1.60000   1.80000
Y:  -5.00000  -3.00000  -2.00000    .00000   3.00000
ERROR SUM OF SQUARE =    .11000E+01
Y - INTERCEPT         =  -.14700E+02
SLOPE                 =    .95000E+01
PREDICTION EQUATION: Y^=  -14.70000 +    9.50000 *X
```

$\hat{L}(x) = -14.7 + 9.5x$

$E(\hat{L}) = 1.1$

5.2 Find the normal equations for the following guess functions. Are they linear?

(a) $g(x) = \alpha x^{\beta}$

[Note: $dx^{\beta}/d\beta = x^{\beta}\ln x$]

(b) $g(x) = \alpha x/(\beta+x)$

(c) $g(x) = -\alpha \exp(-x) + \beta \exp(-4x)$

(d) $g(x) = \alpha + \beta \sin(\gamma x)$

[Note: The answer is a 3 x 3 systems.]

(a) $g(x) = \alpha x^{\beta}$

$$E(g) = \sum_{k=1}^{m}[g(x_k) - y_k]^2$$

$$= \sum[\alpha x_k^{\beta} - y_k]^2$$

$$\partial E(g)/\partial\alpha = \alpha(\sum x_k^{2\beta}) - \sum y_k x_k^{\beta} = 0$$

$$\partial E(g)/\partial\beta = \alpha(\sum x_k^{2\beta} \ln x_k) - \sum y_k x_k^{\beta} \ln x_k = 0$$

ซึ่งไม่เป็นเชิงเส้น

(b) $g(x) = \alpha x/(\beta+x)$

$$E(g) = \sum_{k=1}^{m}[g(x_k) - y_k]^2$$

$$= \sum[\alpha x_k/(\beta + x_k) - y_k]^2$$

$$\partial E(g)/\partial\alpha = 2\ \Sigma[\alpha x_k/(\beta\ t\ x_k) - y_k][x_k/(\beta\ t\ x_k)] = 0$$

$$\partial E(g)/\partial\beta = 2\ \Sigma[\alpha x_k/(\beta+x_k)-y_k]\ \partial[\alpha x_k/(\beta+x_k)-y_k]/\partial\beta$$

$$= 2\ \Sigma[\alpha x_k/(\beta\ t\ x_k) - y_k]\ \alpha x_k \ln(\beta\ t\ x_k) = 0$$

ซึ่งไม่เป็นเชิงเส้น

(c) $g(x) = \alpha \exp(-x)\ t\ \beta \exp(-4x)$

$$E(g) = \sum_{k=1}[g(x_k) - y_k]^2$$

$$= \Sigma[\alpha \exp(-x_k)\ t\ \beta \exp(-4x_k) - y_k]^2$$

$$\partial E(g)/\partial\alpha = \alpha(\Sigma \exp(2x_k)\ t\ \beta\ \Sigma \exp(-5x_k) - \Sigma y_k \exp(-x_k) = 0$$

$$\partial E(g)/\partial\beta = \alpha(\Sigma \exp(-5x_k)\ t\ \beta\ \Sigma \exp(-8x_k) - \Sigma y_k \exp(-4x_k) = 0$$

ซึ่งเป็นเชิงเส้น

(d) $g(x) = \alpha\ t\ \beta \sin(\gamma x)$

$$E(g) = \sum_{k=1}^{m}[g(x_k) - y_k]^2$$

$$= \Sigma\{[\alpha\ t\ \beta \sin(\gamma x_k)] - y_k\}^2$$

$$\partial E(g)/\partial\alpha = 2\ \Sigma[\alpha\ t\ \beta \sin(\gamma x_k) - y_k]$$

$$= 2m\alpha\ t\ 2\beta\ \Sigma \sin(\gamma x_k) - 2\ \Sigma y_k = 0$$

$$\partial E(g)/\partial \beta = 2 \sum [\alpha + \beta \sin(\gamma x_k) - y_k] \sin(\gamma x_k)$$
$$= 2\sum \alpha \sin(\gamma x_k) + 2\beta \sum \sin^2(\gamma x_k) - 2\sum y_k \sin(\gamma x_k) = 0$$

$$\partial E(g)/\partial \gamma = 2 \sum [\alpha + \beta \sin(\gamma x_k) - y_k] \beta \cos(\gamma x_k) x_k$$
$$= 2\beta\alpha \sum x_k \cos(\gamma x_k) + 2\beta^2 \sum x_k \sin(\gamma x_k)\cos(\gamma x_k) - 2\beta \sum x_k y_k \cos(\gamma x_k) = 0$$

ซึ่งไม่เป็นเชิงเส้น

5.3 (a) Determine graphically from Figure 5.2-l which of $g(x) = \alpha \exp(\beta x)$ or $h(x) = \alpha x^{\beta}$ seems best suited to fit the following **data**:

$P_1(1,2.3), P_2(2,6.1), P_3(3,10.7), P_4(4,16.0), P_5(5,21.9), P_6(6,28.3)$

(b) Use the Linearization Algorithm to fit $g(x)$ and $h(x)$ to the data, and find $E(g)$ and $E(h)$. Do your results confirm your answer to part (a)?

Note: $E(g) = E[g(x)] = \sum_{k=1}^{m} [g(x_k) - y_k]^2$

$E(h) = E[h(x)] = \sum_{k=1}^{m} [h(x_k) - y_k]^2$

Ans. $g(x) \doteq 1.9942 \exp(0.47962\ x)$

$h(x) \doteq 2.3032\ x^{1.3005}$

(c) Find $E(\hat{L})$ for the linearized data [i.e., $Q_k(X_k, Y_k)$] for $g(x)$ and for $h(x)$. Do your results confirm your answer to part (a)?

Note: $E(\hat{L}) = \sum_{k=1}^{m} [\hat{a} + \hat{b} X_k - Y_k]^2$

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```
C*****CALLING PROGRAM FOR LINFIT****
C*****PROBLEM 5.3 OR 205*****
      DJMENSION X(6),Y(6),G(6),H(6)
      REAL LNY(6),LNX(6)
      DATA X,Y/1.,2.,3.,4.,5.,6.,2.3,6.1,10.7,16.0,21.9,28.3/
      OPEN(6,FILE='B:RLINFIT1.OUT',STATUS='NEW')
      N=6
      DO 10 I=1,6
      LNY(I)=ALOG(Y(I))
10    LNX(I)=ALOG(X(I))
      CALL LINFIT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL  PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL LINFIT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      ALPHA=EXP(YCEPT)
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
11    FORMAT(1X,'ALPHA = ',F10.5,' BETA = ',F10.5)
      EG=0
      DO 20 I=1,6
      G(I)=ALPHA*EXP(BETA*X(I))
20    EG=EG+(G(I)-Y(I))**2
      WRITE(6,12)EG
12    FORMAT(1X,'E[g(x)]=',E12.7)
      CALL LINFIT(N,LNX,LNY,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,LNX,LNY,SQUERR,YCEPT,SLOPE)
      ALPHA=EXP(YCEPT)
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
      EH=O
      DO 30 I=1,6
      H(I)=ALPHA*X(I)**BETA
30    EH=EH+(H(I)-Y(I))**2
      WRITE(6,13)EH
13    FORMAT(1X,'E[h(x)]=',E12.7)
      STOP
      END
C*****SUBROUTINE TO PRINT OUTPUT*****
      SUBROUTINE PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      DIMENSION X(20),Y(20)
      WRITE(6,2)(X(I),I=1,N)
2     FORMAT(/' X:',6F10.5)
      WRITE(6,3)(Y(I),I=1,N)
3     FORMAT(' Y:',6F10.5)
      WRITE(6,4)SQUERR,YCEPT,SLOPE
4     FORMAT(' ERROR SUM OF SQUARE E[L^] = ',E14.7/
     t       ' Y - INTERCEPT             = ',E14.7/
     t       ' SLOPE                     = ',E14.7)
      WRITE(6,5)YCEPT,SLOPE
5     FORMAT(' PREDICTION EQUATION:  Y^= ',F8.5,' +',F8.5,' *X    ')
      RETURN
      END
```

```
X: 1.00000  2.00000   3.00000   4.00000   5.00000   6.00000
Y:  2.30000  6.10000  10.70000  16.00000  21.90000  28.30000
ERROR SUM OF SQUARE E[L^] =   .3961378E+01
Y - INTERCEPT             =  -.4053333E+01
SLOPE                     =   .52220000E+01
PREDICTION EQUATION: Y^= -4.05333 t 5.22000 *X

X:   1.00000  2.00000   3.00000   4.00000   5.00000   6.00000
Y:   .83291   1.80829   2.37024   2.77259   3.08649   3.34286
ERROR SUM OF SQUARE E[L^] =   .2744317E+00
Y - INTERCEPT             =   .6902263E+00
SLOPE                     =   .4796201E+00
PREDICTION EQUATION:  Y^=   .69023 t .47962 *X
ALPHA =    1.99417  BETA  =    .47962
E[g(x)]=.6379251E+02

X:  .00000   .69315   1.09861   1.38629   1.60944   1.79176
Y:   .83291   1.80829   2.37024   2.77259   3.08649   3.34286
ERROR SUM OF SQUARE E[L^] =   .2519730E-04
Y - INTERCEPT             =   .8342940E+00
SLOPE                     =   .1399493E+01
PREDICTION EQUATION:  Y^=   .83429 t 1.39949 *X
ALPHA  =  2.30319  BETA =  1.39949
E[h(x)]=.2546936E-02
```

5.4 Do (a)-(c) of the Exercise 5.3 for $g(x) = \alpha/(\beta+x)$,

$h(x) = \alpha\, x/(\beta+x)$, and $P_1(0.1,0.04)$, $P_2(1,0.51)$, $P_3(2,1.2)$,

$P_4(3,2.2)$, $P_5(4,3.8)$, $P_6(6,13.2)$.

Ans. (b) $g(x) \doteq -4.5772/(-6.2710+x)$;

$h(x) \doteq -3.1829\ x/(-7.4254\ t\ x)$

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```
C*****CALLING PROGRAM FOR LINFIT****
C*****PROBLEM 5.4 OR 205*****
      DIMENSION X(6),Y(6),G(6),H(6)
      REAL XY(6),DYX(6)
      DATA X,Y/0.1,1.,2.,3.,4.,6.,.04,.51,1.2,2.2,3.8,13.2/
      OPEN(6,FILE='A:RLINFIT2.OUT',STATUS='NEW')
      N=6
      DO 10 I=1,6
      XY(I)=X(I)*Y(I)
10    DYX(I)=Y(I)/X(I)
      CALL     LINFIT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL     PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL LINFIT(N,XY,Y,SQUERR,YCEPT,SLOPE)
      CALL     PRINT(N,XY,Y,SQUERR,YCEPT,SLOPE)
      ALPHA=-YCEPT/SLOPE
      BETA=-1./SLOPE
      WRITE(6,11)ALPHA,BETA
11    FORMAT(1X,'ALPHA = ',F10.5,' BETA = ',F10.5)
      EG=0
      DO 20 I=1,6
      G(I)=ALPHA/(BETAtX(I))
20    EG=EG+(G(I)-Y(I))**2
      WRITE(6,12)EG
12    FORMAT(1X,'E[g(x)]=',E12.7)
      CALL     LINFIT(N,DYX,Y,SQUERR,YCEPT,SLOPE)
      CALL     PRINT(N,DYX,Y,SQUERR,YCEPT,SLOPE)
      ALPHA=YCEPT
      BETA=-SLOPE
      WRITE(6,11)ALPHA,BETA
      EH=0
      DO 30 I=1,6
      H(I)=(ALPHA*X(I))/(BETA+X(I))
30    EH=EHt(H(I)-Y(I))**2
      WRITE(6,13)EH
13    FORMAT(1X,'E[h(x)]=',E12.7)
      STOP
      END
```

X: .10000 1.00000 2.00000 3.00000 4.00000 6.00000
Y: .04000 ‘51000 1.20000 2.20000 3.80000 13.20000
ERROR SUM OF SQUARE [E(L^)] = .2232199E+02
Y - INTERCEPT = -.2119882E+01
SLOPE = ..2091260E+01
PREDICTION EQUATION: Y^= -2.11988 t 2.09126 *X

X: .00400 .51000 2.40000 6.60000 15.20000 79.20000
Y: .04000 .51000 1.20000 2.20000 3.80000 13.20000
ERROR SUM OF SQUARE [E(L^)] = .1192706E+01
Y - INTERCEPT = . 7298974Et00
SLOPE = .1594647E+00
PREDICTION EQUATION: Y^= .72990 t .15946 *X
ALPHA = -4.57717 BETA = -6.27098
E[g(x)]=.1808747E+02

X: .40000 .51000 .60000 .73333 .95000 2.20000
Y: .04000 .51000 1.20000 2.20000 3.80000 13.20000
ERROR SUM OF SQUARE [E(L^)] = .8913414E-01
Y - INTERCEPT ▫ -.3182933E+01
SLOPE = ..7425389E+01
PREDICTION EQUATION: Y^= -3.18293 t 7.42539 *X
ALPHA = -3.18293 BETA = -7.42539
E[h(x)]=.4890339E-01

5.5 Do (a)-(c) of Exercise 5.3 for $g(x) = \alpha+\beta \ln x$, $h(x) = \alpha+(\beta/x)$, and $P_1(1,0.2), P_2(2,1.8), P_3(3,2.6), P_4(5,3.8), P_5(7,4.5), P_6(10,5.3)$.

Ans. (b) $g(x) \doteq 0.21887 + 2.2075 \ln x$;

$h(x) \doteq 5.0403 - 5.2902 / x$

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```
C*****CALLING PROGRAM FOR LINFIT****
C*****PROBLEM 5.5 OR 205*****
      DIMENSION X(6),Y(6),G(6),H(6)
      REAL LNX(6),RX(6)
      DATA X,Y/1.,2.,3.,5.,7.,10.,.2,1.8,2.6,3.8,4.5,5.3/
      OPEN(6,FILE='A:RLINFIT3.OUT',STATUS='NEW')
      N=6
      DO 10 I=1,6
      LNX(I)=ALOG(X(I))
10    RX(I)=1./X(I)
      CALL LINFIT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL LINFIT(N,LNX,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,LNX,Y,SQUERR,YCEPT,SLOPE)
      ALPHA=YCEPT
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
11    FORMAT(1X,'ALPHA = ',F10.6,'  BETA = ',F10.6)
      EG=0
      DO 20 I=1,6
      G(I)=ALPHA+BETA*ALOG(X(I))
20    EG=EG+(G(I)-Y(I))**2
      WRITE(6,12)EG
12    FORMAT(1X,'E[g(x)]=',E12.7)
      CALL LINFIT(N,RX,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,RX,Y,SQUERR,YCEPT,SLOPE)
      ALPHA=YCEPT
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
      EH=0
      DO 30 I=1,6
      H(I)=ALPHA+BETA/X(I)
30    EH=EH+(H(I)-Y(I))**2
      WRITE(6,13)EH
13    FORMAT(1X,'E[h(x)]=',E12.7)
      STOP
      END
```

```
X:   1.00000   2.00000   3.00000    5.00000   7.00000   10.00000
Y:  .20000  1.80000   2.60000    3.80000   4.50000   5.30000
ERROR  SUM  OF  SQUARE  E[L^] =    .1740757E+01
Y  -  INTERCEPT                =    .5779068E+00
SLOPE                          =    .5261628E+00
PREDICTION   EQUATION:  Y^=     .577907 t .526163  *X

X:  ,  00000  .69315    1.09861   1.60944   1.94591   2.30259
Y:  .20000  1.80000   2.60000   3.80000   4.50000   ~5.30000
ERROR SUM OF SQUARE  E[L^ ] =   .5913733E-02
Y - INTERCEPT                 =  .  2188726E+00
SLOPE                         =   . 2207509E+01
PREDICTION EQUATION: Y^=     .218873  t  2.207509   *X
ALPHA  =    .218873  BETA  =    2.207509
E[g(x)]=.5912100E-02

X:   1  .00000  .50000    .33333    .20000    .14286    .10000
Y  :  20000  1.80000   2.60000   3.80000   4.50000   5.30000
ERROR SUM OF SQUARE E[L^] =  .  1716579E+01
Y - INTERCEPT                =    .5040266E+01
SLOPE                        =   -.5290241E+01
PREDICTION EQUATION: Y^=    5.040266  t  -5.290241   *X
ALPHA  =  5.040266  BETA =  -5.290241
E[h(x)]=.1716581E+01
```

5.6 Do (a)-(c) of Exercise 5.3 for $g(x) = \alpha/(\beta+x)$,

$h(x) = \alpha \exp(\beta x)$, and $P_1(1,0.9), P_2(2,2.2), P_3(3,5.4)$,

$P_4(4,13.2), P_5(5,32.6), P_6(6,77.4)$.

Ans. (b) $g(x) \doteq -17.499/(-6.1252+x)$;

$h(x) \doteq 0.36979 \exp(0.69295\ x)$

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```
C*****CALLING PROGRAM FOR LINFIT****
C*****PROBLEM 5.6 OR 205*****
      DIMENSION X(6),Y(6),G(6),H(6)
      REAL XY(6),LNY(6)
      DATA X,Y/1.,2.,3.,4.,5.,6.,.9,2.2,5.4,13.2,32.6,77.4/
      OPEN(6,FILE='A:RLINFIT4.OUT',STATUS='NEW')
      N=6
      DO 10 I=1,6
      XY(I)=X(I)*Y(I)
10    LNY(I)=ALOG(Y(I))
      CALL LINFIT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,X,Y,SQUERR,YCEPT,SLOPE)
      CALL LINFIT(N,XY,Y,SQUERR,YCEPT,SLOPE)
      CALL PRINT(N,XY,Y,SQUERR,YCEPT,SLOPE)
      ALPHA=-YCEPT/SLOPE
      BETA=-1 ./SLOPE
      WRITE(6,11)ALPHA,BETA
11    FORMAT(1X,'ALPHA = ',F10.6,'  BETA = ',F10.6)
      EG=0
      DO 20 I=1,6
      G(I)=ALPHA/(BETA+X(I))
20    EG=EG+(G(I)-Y(I))**2
      WRITE(6,12)EG
12    FORMAT(1X,'E[g(x)]=',E12.7)
      CALL LINFIT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      CALL,PRINT(N,X,LNY,SQUERR,YCEPT,SLOPE)
      ALPHA=EXP(YCEPT)
      BETA=SLOPE
      WRITE(6,11)ALPHA,BETA
      EH=0
      DO 30 I=1,6
      H(I)=ALPHA*EXP(BETA*X(I))
30    EH=EHt(H(I)-Y(I))**2
      WRITE(6,13)EH
13    FORMAT(1X,'E[h(x)]=',E12.7)
      STOP
      END
```

```
X:    1.00000   2.00000   3.00000   4.00000   5.00000   6.00000
Y:     .90000   2.20000   5.40000  13.20000  32.60000  77.40000
ERROR SUM OF SQUARE E[L^] =    .1059723E+04
Y - INTERCEPT              =  -.2620000E+02
SLOPE                      =    .13735714E+02
PREDICTION EQUATION: Y^= -26.200000 t 13.757140 *X


X:     .90000   4.40000  16.20000  52.80000 163.00000 464.40000
Y:     .90000   2.20000   5.40000  13.20000  32.60000  77.40000
ERROR SUM OF' SQUARE E[L^] =    .2072951E+02
Y - INTERCEE'T             =    .2856798E+01
SLOPE                      =  . 1632595E+00
PREDICTION EQUATION: Y^=   2.856798 t .163260 *X
ALPHA = -17.498510 BETA = -6.125217
E[g(x)]=.4212815E+04

X: 1.00000   2.00000   3.00000   4.00000   5.00000   6.00000
Y  '  -.10536   .78846   1.68640   2.58022   3.48431   3.34899
ERROR SUM OF SQUARE E[L^] =    .4351788E-03
Y - INTERCEE'T             =  -.9948099E+00
SLOPE                      =    .8929462E+00
PREDICTION EQUATION: Y^=  -.994810 t .892946 *X
ALPHA =    .369794 BETA =    .892946
E[h(x)]=.1389400E+01
```