3.1 Consider the 2x2 linear system: $x+2y = -3$; $4x+3y = 18$

(a) Form A, x, b, and [A:b] for the matrix form Ax = b.

(b) Find A^{-1} and verify that $\bar{x} = A^{-1}b$ is a solution.

(a) $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \end{bmatrix}$, $b = \begin{bmatrix} -3 \\ 18 \end{bmatrix}$

$[A:b] = \begin{bmatrix} 1 & 2 & : & -3 \\ 4 & 3 & : & 18 \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} -3/5 & 2/5 \\ 4/5 & -1/5 \end{bmatrix}$ ดังนั้น $\bar{x} = A^{-1}b = \begin{bmatrix} 9 \\ -6 \end{bmatrix}$

3.2 Do parts (a) and (b) of Exercise 3.1 for: $-2x+y = -1$; $3x-y = 3$.

(a) $A = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$[A:b] = \begin{bmatrix} -2 & 1 & : & -1 \\ 3 & -1 & : & 3 \end{bmatrix}$

(b) $A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ ดังนั้น $\bar{x} = A^{-1}b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

3.3 Use Forward Substitution to complete $[L:b:\bar{c}]$.

(a) $\left[\begin{array}{ccc:c:c} 1 & 0 & 0 & 3 & \\ 2 & 3 & 0 & 0 & \\ 4 & 5 & 6 & 4 & \end{array}\right]$ (b) $\left[\begin{array}{ccc:c:c} 6 & 0 & 0 & -6 & \\ 5 & 4 & 0 & 3 & \\ 3 & 2 & 1 & 0 & \end{array}\right]$

$\quad L \qquad b \qquad \bar{c} \qquad\qquad\qquad L \qquad b \qquad \bar{c}$

(c) $\left[\begin{array}{cccc:c:c} 2 & & & & -2 & \\ 0 & 1 & & & -1 & \\ -2 & 0 & 3 & & 2 & \\ 0 & 1 & 1 & 2 & -1 & \end{array}\right]$

$\quad L \qquad b \qquad \bar{c}$

Verify that $\bar{c}$ obtained [in (a), (b) and (c)] satisfies $L\bar{c} = b$.

3.3 (a) $\left[\begin{array}{ccc:c:c} 1 & 0 & 0 & 3 & 3 \\ 2 & 3 & 0 & 0 & -2 \\ 4 & 5 & 6 & 4 & 1/3 \end{array}\right]$ $\longrightarrow$

$\bar{c}_1 = 3$

$\bar{c}_2 = (1/3)\{0-[2][3]\} = -2$

$\bar{c}_3 = (1/6)\{4-[4\ 5]\begin{bmatrix} 3 \\ -2 \end{bmatrix}\} = 1/3$

$\quad L \qquad b \qquad \bar{c}$

ตรวจสอบว่า $L\bar{c} = b$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 3 \\ -2 \\ 1/3 \end{bmatrix}}_{\bar{c}} = \underbrace{\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}}_{b}$$

3.3 (b)

$$\left[\begin{array}{ccc:c:c} 6 & & & -6 & -1 \\ 5 & 4 & & 3 & 2 \\ 3 & 2 & 1 & 0 & -1 \end{array}\right] \quad \begin{array}{l} L \quad b \quad \bar{c} \end{array}$$

$\dashrightarrow \bar{c}_1 = -6/6 = -1$

$\bar{c}_2 = (1/4)\{3-[5][-1]\} = 2$

$\bar{c}_3 = \{0-[3\ 2]\begin{bmatrix} -1 \\ 2 \end{bmatrix}\} = -1$

ตรวจสอบว่า $L\bar{c} = b$

$$\underbrace{\begin{bmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}}_{\bar{c}} = \underbrace{\begin{bmatrix} -6 \\ 3 \\ 0 \end{bmatrix}}_{b}$$

3.3(c)

$$\left[\begin{array}{cccc:c:c} 2 & 0 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ -2 & 0 & 3 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \end{array}\right] \quad \begin{array}{l} L \quad b \quad \bar{c} \end{array}$$

$\dashrightarrow \bar{c}_1 = -2/2 = -1$

$\bar{c}_2 = \{-1-[0][-1]\} = -1$

$\bar{c}_3 = (1/3)\{2-[-2\ 0]\begin{bmatrix} -1 \\ -1 \end{bmatrix}\} = 0$

$\bar{c}_4 = (1/2)\{-1-[0\ 1\ 1]\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}\}$

$= 0$

ตรวจสอบว่า $L\bar{c} = b$

$$
\underset{L}{\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}}
\underset{\bar{c}}{\begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}}
=
\underset{b}{\begin{bmatrix} -2 \\ -1 \\ 2 \\ -1 \end{bmatrix}}
$$

3.4 Use Backward Substitution to complete $[U:\bar{c}:\bar{x}]$.

(a)
$$
\begin{bmatrix} 6 & 5 & 4 & : & 4 & : & \\ & 3 & 2 & : & 0 & : & \\ & & 1 & : & 3 & : & \\ & U & & & \bar{c} & & \bar{x} \end{bmatrix}
$$

(b)
$$
\begin{bmatrix} 1 & 5 & -3 & : & -10 & : & \\ & 1 & -2 & : & -5 & : & \\ & & 1 & : & 2 & : & \\ & U & & & \bar{c} & & \bar{x} \end{bmatrix}
$$

(c)
$$
\begin{bmatrix} 1 & 2 & 3 & 4 & : & 5 & : & \\ & 1 & 5 & 6 & : & 4 & : & \\ & & 1 & 7 & : & -5 & : & \\ & & & 1 & : & -1 & : & \\ & U & & & & \bar{c} & & \bar{x} \end{bmatrix}
$$

Verify that the $\bar{x}$ obtained Cin a), b) and c)] satisfies $U\bar{x} = \bar{c}$.

3.4(a)
$$
\begin{bmatrix} 6 & 5 & 4 & : & 4 & : & 1/3 \\ & 3 & 2 & : & 0 & : & -2 \\ & & 1 & : & 3 & : & 3 \\ & U & & & \bar{c} & & \bar{x} \end{bmatrix}
$$

$\bar{x}_1 = (1/6)\{4-[5\ 4]\begin{bmatrix} -2 \\ 3 \end{bmatrix}\} = 1/3$

$x_2 = (1/3)\{0-[2][3]\} = 3 = -2$

$\bar{x}_3 = \bar{c}_3 = 3$

ตรวจสอบว่า $U\bar{x} = \bar{c}$

$$\underset{U}{\begin{bmatrix} 6 & 5 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}} \underset{\bar{x}}{\begin{bmatrix} 1/3 \\ -2 \\ 3 \end{bmatrix}} = \underset{\bar{c}}{\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}}$$

3.4(b)

$$\left[\begin{array}{ccc:c:c} 1 & 5 & -3 & -10 & 1 \\ & 1 & -2 & -5 & -1 \\ & & 1 & 2 & 2 \\ & U & & \bar{c} & \bar{x} \end{array}\right] \dashrightarrow \begin{aligned} \bar{x}_1 &= -10-[5 \quad -3]\begin{bmatrix} -1 \\ 2 \end{bmatrix} = 1 \\ \bar{x}_2 &= -5-[-2][2] = -1 \\ \bar{x}_3 &= 2 \end{aligned}$$

ตรวจสอบว่า $U\bar{x} = \bar{c}$

$$\underset{U}{\begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}} = \underset{\bar{c}}{\begin{bmatrix} -10 \\ -5 \\ 2 \end{bmatrix}}$$

3.4(c)

$$\left[\begin{array}{cccc:c:c} 1 & 2 & 3 & 4 & 5 & 3 \\ & 1 & 5 & 6 & 4 & 0 \\ & & 1 & 7 & -5 & 2 \\ & & & 1 & -1 & -1 \\ & U & & & \bar{c} & \bar{x} \end{array}\right] \dashrightarrow \begin{aligned} \bar{x}_1 &= 5-[2 \quad 3 \quad 4]\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = 3 \\ \bar{x}_2 &= 4-[5 \quad 6]\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0 \\ \bar{x}_3 &= -5-[7][-1] = 2 \\ \bar{x}_4 &= -1 \end{aligned}$$

ตรวจสอบว่า $U\bar{x} = \bar{c}$

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} 3 \\ 0 \\ 2 \\ -1 \end{bmatrix}}_{\bar{x}} = \underbrace{\begin{bmatrix} 5 \\ 4 \\ -5 \\ -1 \end{bmatrix}}_{\bar{c}}$$

3.5 For the A matrices given in a)-d), show that the Triangular Decomposition Algorithm with (i) Basic Pivoting and (ii) Partial Pivoting yields the indicated LU-Decomposition.
Find $\hat{A}$ in a)-d) for (i) and (ii). ($\hat{A}$ คือ A ถ้าไม่ต้องทำ row interchange)

หมายเหตุ ข้อ (ii) นักศึกษาต้องทำจาก A แล้วเลือก pivot ตามวิธีของ PP (ไม่ใช่หา $\hat{A}$ แล้วหา $\hat{L}\backslash\hat{U}$)

3.5(a) $A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$

(i) Basic Pivoting

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 2 & 1/2 \\ 2 & 1 & 5/2 \end{bmatrix} \quad \rho_1 \rightleftarrows \rho_2$$

(ii) Partial Pivoting

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 2 & -1/2 & 3/2 \\ 0 & 2 & 1/2 \\ -1 & 1/2 & 5/4 \end{bmatrix} \quad \begin{matrix} \rho_1 \rightleftarrows \rho_3 \\ \rho_2 \rightleftarrows \rho_3 \end{matrix}$$

3.5(b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 0 & -1 & 3 \end{bmatrix}$$

( ) Basic Pivoting

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -5 & 0 \\ 0 & 1 & 3 \end{bmatrix} = L\backslash U$$

(ii) Partial Pivot ing

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 2 & -1/2 & 1 \\ 1 & 5/2 & 0 \\ 0 & 1 & 3 \end{bmatrix} \rho_1 \rightleftarrows \rho_2$$

3.5(c)

$$A = \begin{bmatrix} 0 & 2 & 1 & 4 \\ 1 & -1 & -2 & 0 \\ 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \end{bmatrix}$$

(i) Basic Pivoting

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 2 & 1/2 & 2 \\ 2 & 2 & 3 & 2/3 \\ -1 & 2 & -1 & -10/3 \end{bmatrix} \rho_1 \rightleftarrows \rho_2$$

(ii) Partial Pivoting

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 2 & 0 & 0 & 3 \\ -1 & 3 & 2/3 & 1 \\ 1 & -1 & -4/3 & 3/2 \\ 0 & 2 & -1/3 & 5/2 \end{bmatrix} \begin{matrix} \rho_1 \rightleftarrows \rho_3 \\ \rho_2 \rightleftarrows \rho_4 \\ \rho_3 \rightleftarrows \rho_4 \\ \end{matrix}$$

3.5 (d)

$$A = \begin{vmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & 4 \\ 3 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{vmatrix}$$

(i) **Basic Pivoting** (ii) **Partial Pivoting**

$$L\backslash U = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & 2 \\ 3 & 0 & 5 & -3/5 \\ 0 & -1 & -1 \quad 7/5 & 7/5 \end{bmatrix} = L\backslash U \qquad \hat{L}\backslash\hat{U} = \begin{bmatrix} 3 & 0 & -1/3 & 0 \\ 0 & -5/3 & & \\ 1 & 0 & -5/3 & -3/5 \\ 0 & -1 & -1 & 7/5 \\ & 2 & 0 & 2 \end{bmatrix} \rho_1 \rightleftarrows \rho_3$$

AA

(a), (b) และ (d) ให้นักศึกษาทำเอง

3.5 (c) i) **Basic Pivoting**

Phase 1 **Phase 2**

$$A = \begin{bmatrix} 1 & -1 & -2 & 0 \\ 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \end{bmatrix} 5 \dashrightarrow \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 2 & & 14 \\ 2 & 0 & & 06 \\ -1 & 3 & 2 & 0 \end{bmatrix} = \hat{A}$$

$$L\backslash U = \begin{bmatrix} 0 & & & \\ 1 & & & \\ 2 & & & \\ -1 & & & \end{bmatrix} \dashrightarrow \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 2 & 1/2 & 2 \\ 2 & 2 & 3 & 2/3 \\ -1 & 2 & -1 & -10/3 \end{bmatrix} = \hat{L}\backslash\hat{U}$$

$m = 1$: $col_1 \hat{L} = col_1 A$ = CO 0 1 2 -11'

สลับ row 1 และ row 2 (ทำต่อ Phase 2)

$[\hat{u}_{12} \quad \hat{u}_{13} \quad \hat{u}_{14}] = (1/1)[-1 \quad -2 \quad 0]$

$m = 2$: $\hat{l}_{22} = 2-[0][-1] = 2$

$\hat{l}_{32} = 0-[2][-1] = 2$

$\hat{l}_{42} = 3-[-1][-1] = 2$

$\hat{u}_{23} = (1/2)(1-[0][-2]) = 1/2$

$\hat{u}_{24} = (1/2)(4-[2][0]) = 2$

$m = 3$: $\hat{l}_{33} = 0 - [2 \quad 2]\begin{bmatrix} -2 \\ 1/2 \end{bmatrix} = 3$

$\hat{l}_{43} = 2 - [-1 \quad 2]\begin{bmatrix} \\ 1/2 \end{bmatrix} = -1$

$\hat{u}_{34} = (1/3)(6 - [2 \quad 2]\begin{bmatrix} 0 \\ 2 \end{bmatrix}) = 2/3$

$m = 4$: $\hat{l}_{44} = 0 - [-1 \quad 2 \quad -1]\begin{bmatrix} 0 \\ 2 \\ 2/3 \end{bmatrix} = -10/3$

3.5 (c) ii) Partial Pivoting

Phase 1	Phase 2	Phase 3	Phase 4

$$A = \begin{bmatrix} 0 & 2 & 1 & 4 \\ 1 & -1 & -2 & 0 \\ 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 6 \\ 1 & -1 & -2 & 0 \\ 0 & 2 & 1 & 4 \\ -1 & 3 & 2 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \\ 0 & 2 & 1 & 4 \\ 1 & -1 & -2 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \\ 1 & -1 & -2 & 0 \\ 0 & 2 & 1 & 4 \end{bmatrix} = \hat{A}$$

$$L\backslash U = \begin{bmatrix} 0 & & & \\ 1 & & & \\ 2 & & & \\ -1 & & & \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 3 \\ 1 & -1 & & \\ 0 & 2 & & \\ -1 & 3 & & \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 3 \\ -1 & 3 & 2/3 & 1 \\ 0 & 2 & -1/3 & \\ 1 & -1 & -4/3 & \end{bmatrix} \dashrightarrow \begin{bmatrix} 2 & 0 & 0 & 3 \\ -1 & 3 & 2/3 & 1 \\ 1 & -1 & -4/3 & 3/2 \\ 0 & 2 & -1/3 & 5/2 \end{bmatrix}$$

$$= \hat{L}\backslash\hat{U}$$

$m = 1$: $\mathrm{col}_1\hat{L} = \mathrm{col}_1 A = [0 \quad 1 \quad 2 \quad -1]'$

สลับ row 1 และ row 3 (ทำต่อ Phase 2)

$$[\hat{u}_{12} \quad \hat{u}_{13} \quad \hat{u}_{14}] = (1/2)[0 \quad 0 \quad 6] = [0 \quad 0 \quad 3]$$

$m = 2$: $\hat{l}_{22} = -1-[0][0] = -1$

$\hat{l}_{32} = 2-[0][0] = 2$

$\hat{l}_{42} = 3-[-1][0] = 3$

สลับ row 2 และ row 4 (ทำต่อ Phase 3)

$\hat{u}_{23} = (1/3)(2-[-1][0]) = 2/3$

$\hat{u}_{24} = (1/3)(0-[-1][3]) = 1$

m = 3: $\hat{l}_{33} = 1 - [0 \quad 2]\begin{bmatrix}0\\2/3\end{bmatrix} = -1/3$

$\hat{l}_{43} = -2 - [1 \quad -1]\begin{bmatrix}0\\2/3\end{bmatrix} = -4/3$

สลับ row 3 และ row 4 (ทำต่อ Phase 4)

$\hat{u}_{34} = [1/(-4/3)](0 - [1 \quad -1]\begin{bmatrix}3\\1\end{bmatrix}) = 3/2$

m = 4: $\hat{l}_{44} = 4 - [0 \quad 2 \quad -1/3]\begin{bmatrix}3\\1\\3/2\end{bmatrix} = 5/2$

3.6 Obtain $\hat{A}$ and A from the given LU-Decomposition.

Show work.

$$\hat{L}\backslash\hat{U} = \begin{array}{cl}\begin{bmatrix}2 & 4 & 1\\0 & 2 & 1\\1 & -1 & 1\end{bmatrix} & \begin{array}{l}\rho_1 \rightleftarrows \rho_2\\ \rho_2 \rightleftarrows \rho_3\\ \\ \end{array}\end{array}$$

[Hint: เขียน $\hat{L}$ และ $\hat{U}$ ก่อน แล้วหา $\hat{A} = \hat{L}\hat{U}$ (ไม่ต้องแสดงการหา $\hat{L}\backslash\hat{U}$)

ต่อไปหา A โดยทำ row interchanges in reverse order กับ $\hat{A}$

คือทำ $(\rho_2 \rightleftarrows \rho_3)$ ก่อน แล้วทำ $(\rho_1 \rightleftarrows \rho_2)$]

$$\hat{L} = \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\1 & -1 & 1\end{bmatrix}, \quad \hat{U} = \begin{bmatrix}1 & 4 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$$

$$\hat{A} = \hat{L}\hat{U} = \begin{vmatrix} 2 & 8 & 2 \\ 0 & 2 & 2 \\ 1 & 3 & 1 \end{vmatrix} \longrightarrow \begin{vmatrix} 2 & 8 & 2 \\ 1 & 3 & 1 \\ 0 & 2 & 2 \end{vmatrix} \longrightarrow \begin{vmatrix} 1 & 3 & 1 \\ 2 & 8 & 2 \\ 0 & 2 & 2 \end{vmatrix} = A$$

3.7 จากเมตริกซ์ A ใน (a)-(d) ของข้อ 3.5 จงหา det A โดยใช้สูตร D8 สำหรับ $\hat{L}\backslash\hat{U}$ จากทั้งสอง pivoting strategies [จากทั้ง (i) และ (ii)]

จากสูตร

$$\boxed{\begin{array}{l} \det A = (-1)^{\rho}[\text{product of the pivots of } \hat{L}\backslash\hat{U}] \\ \rho = \text{จำนวน row interchanges ที่ทำให้ได้ } \hat{L}\backslash\hat{U} \end{array}}$$

(a) (i) $\det A = (-1)^1(-1)(2)(5/2) = 5$

(ii) $\det A = (-1)^2(2)(2)(5/4) = 5$

(b) (i) $\det A = (-1)^0(1)(-5)(3) = -15$

(ii) $\det A = (-1)^1(2)(5/2)(3) = -15$

(c) (i) $\det A = (-1)^1(1)(2)(3)(-10/3) = 20$

(ii) $\det A = (-1)^3(2)(3)(-4/3)(5/2) = 20$

(d) (i) $\det A = (-1)^0(1)(2)(5)(7/5) = 14$

(ii) $\det A = (-1)^1(3)(2)(-5/3)(7/5) = 14$

3.8 จงหา A^{-1} ของเมตริกซ์ A ใน a)-d) ของข้อ 3.5 โดยใช้วิธีการใน ข้อ 3.6C โดยใช้ $\hat{L}\backslash\hat{U}$ จาก Partial pivoting เมื่อได้ A^{-1} แล้วแสดงการตรวจสอบด้วย (นั่นคือแสดงว่า $AA^{-1} = I_n$)

3.8 (a)

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$$

จาก (a) ii) ในข้อ 3.5

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 2 & -1/2 & 3/2 \\ 0 & 2 & 1/2 \\ -1 & 1/2 & 5/4 \end{bmatrix} \quad \begin{matrix} r_1 \rightleftarrows r_3 \\ r_2 \rightleftarrows r_3 \end{matrix}$$

สร้าง Forback matrix $[\hat{L}\backslash\hat{U} : \hat{I}_3 : \bar{C} : \bar{X}]$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \hat{I}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Forback matrix $[\hat{L}\backslash\hat{U} : \hat{I}_3 : \bar{C} : \bar{X}]$ คือ

$$\left[\begin{array}{ccc:ccc:ccc:ccc} 2 & -1/2 & 3/2 & 0 & 0 & 1 & 0 & 0 & 1/2 & 3/5 & -7/5 & -1/5 \\ 0 & 2 & 1/2 & 1 & 0 & 0 & 1/2 & 0 & 0 & 3/5 & -2/5 & -1/5 \\ -1 & 1/2 & 5/4 & 0 & 1 & 0 & -1/5 & 4/5 & 2/5 & -1/5 & 4/5 & 2/5 \end{array}\right]$$

$row_1\bar{C} = (1/2)[0 \;\; 0 \;\; 1] = [0 \;\; 0 \;\; 1/2]$

$row_2\bar{C} = (1/2)\{[1 \;\; 0 \;\; 0]-[0][0 \;\; 0 \;\; 1/2]\} = [1/2 \;\; 0 \;\; 0]$

$$row_3\bar{C} = (1/(5/4))\left\{[0 \;\; 1 \;\; 0]-[-1 \;\; 1/2]\begin{bmatrix} 0 & 0 & 1/2 \\ 1/2 & 0 & 0 \end{bmatrix}\right\}$$

$= (4/5)\{[0 \;\; 1 \;\; 0]-[1/4 \;\; 0 \;\; -1/2]\}$

$= (4/5)[-1/4 \;\; 0 \;\; 1/2] = [-1/5 \;\; 4/5 \;\; 2/5]$

$\text{row}_3\bar{X} = \text{row}_3\bar{C}$ = C-1/5 4/5 2/5]

$\text{row}_2\bar{X}$ = [1/2 0 0]-(1/2)[-1/5 4/5 2/5]

= [1/2 0 0]-c-1/10 2/5 1/5]

= [3/5 -2/5 -1/5]

$$\text{row}_1\bar{X} = \text{CO}\ 0\ 1/2] - [-1/2\ \ 3/2]\begin{bmatrix} 3/5 & -2/5 & -1/5 \\ -1/5 & 4/5 & 2/5 \end{bmatrix}$$

= [3/5 -7/5 -1/5]

$$\text{ดังนั้น}\quad A^{-1} = \begin{bmatrix} 3/5 & -7/5 & -1/5 \\ 3/5 & -2/5 & -1/5 \\ -1/5 & 4/5 & 2/5 \end{bmatrix} = (1/5)\begin{bmatrix} 3 & -7 & -1 \\ 3 & -2 & -1 \\ -1 & 4 & 2 \end{bmatrix}$$

ให้นักศึกษาตรวจสอบว่า $AA^{-1} = I$

3.6 (b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 0 & -1 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/3 & 1/3 & -1/3 \\ 2/5 & -1/5 & 0 \\ -2/15 & 1/15 & 1/3 \end{bmatrix} \quad (1/15)\begin{bmatrix} 5 & 5 & -5 \\ 6 & -3 & 0 \\ -2 & 1 & 5 \end{bmatrix}$$

3.6 (c)

$$A = \begin{bmatrix} 0 & 2 & 1 & 4 \\ 1 & -1 & -2 & 0 \\ 2 & 0 & 0 & 6 \\ -1 & 3 & 2 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{vmatrix} -6/5 & 3/10 & 4/5 & 9/10 \\ 0 & 1/2 & 0 & 1/2 \\ -3/5 & -3/5 & 2/5 & 1/5 \\ 2/5 & -1/10 & -1/10 & -3/10 \end{vmatrix} = (1/10)\begin{bmatrix} -12 & 3 & 6 & 9 \\ 0 & 5 & 0 & 5 \\ -6 & -6 & 4 & 2 \\ 4 & -1 & -1 & -3 \end{bmatrix}$$

3.8 (d)

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & 4 \\ 3 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

จาก (d) ii) ในข้อ 3.5

$$\hat{L}\backslash\hat{U} = \begin{bmatrix} 3 & 0 & -1/3 & 0 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & -5/3 & -3/5 \\ 0 & -1 & -1 & 7/5 \end{bmatrix} \rho_1 \rightleftarrows \rho_3$$

$$\hat{I}_4 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Forback matrix $[\hat{L}\backslash\hat{U} : \hat{I}_4 : \bar{C} : \bar{X}]$ **is**

3	0	-1/3	0 :	0	0	1	0 :	0	0	1/3	0 :	-2/7	1/14	3/7	1/7
0	2 0	0	2 :	0	1	0	0 :	0	1/2	0	0 :	6/7	-3/14	-2/7	-10/7
1	0	-5/3	-3/5 :	1	0	0	0 :	-3/5	0	1/5	0 :	-6/7	3/14	2/7	3/7
0	-1	-1	7/5 :	0	0	0	1 :	-3/7	5/14	1/7	5/7 :	-3/7	5/14	1/7	5/7

I)

$\text{row}_1\bar{C} = (1/3)[0\ 0\ 1\ 0] = [0\ 0\ 1/3\ 0]$

$\text{row}_2\bar{C} = (1/2)\{[0\ 1\ 0\ 0] - [0][0\ 0\ 1/3\ 0]\}$

$= [0\ 1/2\ 0\ 0]$

$\text{row}_3\bar{C} = [1/(-5/3)]\{[1\ 0\ 0\ 0] - [1\ 0]\begin{bmatrix} 0 & 0 & 1/3 & 0 \\ 0 & 1/2 & 0 & 0 \end{bmatrix}\}$

$= (-3/5)[1\ 0\ -1/3\ 0] = [-3/5\ 0\ 1/5\ 0]$

$\text{row}_4\bar{C} = [1/(7/5)]\{[0\ 0\ 0\ 1] - [0\ -1\ -1]\begin{bmatrix} 0 & 0 & 1/3 & 0 \\ 0 & 1/2 & 0 & 0 \\ -3/5 & 0 & 1/5 & 0 \end{bmatrix}\}$

$= (5/7)[-3/5\ 1/2\ 1/5\ 1] = [-3/7\ 5/14\ 1/7\ 5/7]$

$\text{row}_4\bar{X} = \text{row}_4\bar{C} = [-3/7\ 5/14\ 1/7\ 5/7]$

$\text{row}_3\bar{X} = [-3/5\ 0\ 1/5\ 0] - [-3/5][-3/7\ 5/14\ 1/7\ 5/7]$

$= [-3/5\ 0\ 1/5\ 0] - [9/35\ -3/14\ -3/35\ -3/7]$

$= [-6/7\ 3/14\ 2/7\ 3/7]$

$\text{row}_2\bar{X} = [0\ 1/2\ 0\ 0] - [0\ 2]\begin{bmatrix} -6/7 & 3/14 & 2/7 & 3/7 \\ -3/7 & 5/14 & 1/7 & 5/7 \end{bmatrix}$

$= [0\ 1/2\ 0\ 0] - [-6/7\ 10/14\ 2/7\ 10/7]$

$= [6/7\ -3/14\ -2/7\ -10/7]$

$\text{row}_1\bar{X} = [0\ 0\ 1/3\ 0] - [0\ -1/3\ 0]\begin{bmatrix} 6/7 & -3/14 & -2/7 & -10/7 \\ -6/7 & 3/14 & 2/7 & 3/7 \\ -3/7 & 5/14 & 1/7 & 5/7 \end{bmatrix}$

$= [0\ 0\ 1/3\ 0] - [2/7\ -1/14\ -10/105\ -1/7]$

$= [-2/7\ 1/14\ 3/7\ 1/7]$

ดังนั้น

$$A^{-1} = \begin{vmatrix} -2/7 & 1/14 & 3/7 & 1/7 \\ 6/7 & -3/14 & -2/7 & -10/7 \\ -6/7 & 3/14 & 2/7 & 3/7 \\ -3/7 & 5/14 & 1/7 & 5/7 \end{vmatrix} = (1/14) \begin{vmatrix} - 4 & 1 & 6 & 2 \\ 12 & -3 & -4 & -20 \\ -12 & 3 & 4 & 6 \\ -6 & 5 & 2 & 10 \end{vmatrix}$$

ให้นักศึกษาตรวจสอบว่า $AA^{-1} = I$