เฉลยแบบฝึกหัดบทที่ 2

2.1 Let $f(x)=x^5-6x^4+9x^3$, $\bar{x}=0$, $x_0=-0.3$ and for (SEC) $x_{-1}=-0.4$

(a) Find x_1, x_2, x_3 and x_4 by (NR). Verify that the convergence to $\bar{x}$ is linear (i.e., $\Delta x_k \approx C\Delta x_{k-1}$)

(b) Repeat part a) for x_1, x_2, x_3, and x_4 obtain using (SEC).

(a) จากการใช้ซับรูทีน NR ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
C:\FORTRAN>rnr21
File name missing or blank - Please enter name
UNIT 6? con

ENTER NUMSIG,MAXIT,XO(INITIAL GUESS) UNIT 5? con
7 10 -0.3

K= 0 XO=   -.3000000  YO=-.2940300E+00
K= 1  X=   -.2057143  Y =-.8946290E-01 DELTAX=   .9428572E-01
K= 2  X=   -.1399561  Y =-.2702852E-01 DELTAX=   .6575824E-01
K= 3  X=   -.0946503  Y =-.8120621E-02 DELTAX=   .4530575E-01
K= 4  X=   -.0637307  Y =-.2429665E-02 DELTAX=   .3091965E-01
K= 5  X=   -60427777  Y =-.7247563E-03 DELTAX=   .2095298E-01
K= 6  x=   -.0286509  Y =-.2157303E-03 DELTAX=   .1412682E-01
K= 7  X=   -.0191604  Y =-.6411912E-04 DELTAX=   .9490432E-02
K= 8  x=   -.0128005  Y =-.1903811E-04 DELTAX=   .6359899E-02
K= 9  X =  -.0085457  Y =-.5648865E-05 DELTAX=   .4254789E-02
K=10 X =   -.0057025  Y =-.1675317E-05 DELTAX=   .2843194E-02
DESIRED ACCURACY IS NOT EVIDENT IN 10 ITERATIONS
```

k	x_k	$\Delta x_k=-y_k/f'(x_k)$	$\Delta x_k/\Delta x_{k-1}$	$x_{k+1}=x_k+\Delta x_k$
0	-0.3	0.09428572		-0.205714
1	-0.205714	0.06575824	0.70	-0.139956
2	-0.139950	0.04530575	0.69	-0.094650
3	-0.094650	0.03091965	0.69	-0.063731
4	-0.063731		≈ 0.7 ---> linear convergence	

(b) จากการใช้ซับรูทีน SECANT ในหัวข้อ 2.1I ได้ผลดังนี้

```
ENTER DESIRED # SIGNIFICANT DIGITS, IOUT (0-2)UNIT 5? con
72

ENTER TWO INITIAL GUESSES (SECOND CLOSER IF POSSIBLE) -0.4 -0.3
 k          x_k              f(x_k)            Δx_k-1
 1   -. 2340459      -.1340897E+00      .6595411E-01
 2   -.1787517       -.5771171E-01      .5529417E-01
 3   -.1369711       -.2528761E-01      .4178063E-01
 4   -.1043863       -.1096181E-01      .3258478E-01
 5   -.0794531       -.4756416E-02      .2493320E-01
 6   -.0603419       -.2057766E-02      .1911124E-01
 7   -.0457692       -.8894355E-03      .1457265E-01
 a   -.0346752       -.3839574E-03      .1109397E-01
 9   -.0262483       -.1656208E-03      .8426899E-02
I0   -.0198561       -.7139234E-04      .6392287E-02
11   -.0150129       -.3075920E-04      .4843125E-02
12   -.0113467       -.1324738E-04      .3666235E-02
13   -.0085733       -.5703724E-05      .2773442E-02
14   -.0064763       -.2455218E-05      .2096987E-02
15   -.0048914       -.1056692E-05      .1584901E-02
16   -. 0036939      -.4547271E-06      .1197512E-02
17   -.0027892       -.1956641E-06      .9046067E-03
18   -.0021060       -.8418596E-07      .6832280E-03
19   -.0015901       -.3621962E-07      .5159543E-03
20   -.0012005       -.1558225E-07      .3896035E-03

DESIRED ACCURACY IS NOT EVIDENT IN 20ITERATIONSStop - Program terminated.
```

k	x_k	$\Delta x_k=(-y_k\Delta x_{k-1})/(y_k-y_{k-1})$	$x_{k+1}=x_k+\Delta x_k$
-1	-0.4		
0	-0.3	0.06595411	-0.2340459
1	-0.234046	0.05529417	-0.1787517
2	-0.178752	0.04178063	-0.1369711
3	-0.136971	0.03258478	-0.1043863
4	-0.104386		

2.2 Do (a) and (b) of Exercise 2.1 with $f(x)=x^5-6x^4+9x^3$ again, but with $\bar{x}=3$, $x_0=3.06$ and for (SEC) $x_{-1}=3.1$.

(a) จากการใช้ซับรูทีน NR ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER NUMSIG,MAXIT,XO(INITIAL GUESS) 7 15 3.0001

K= 0  XO=   3.0001000  YO= .2694731E-06
K= 1  X=    3.0000500  Y = .6768685E-07  DELTAX=  -.4994620E-04
K= 2  X =   3.0000250  Y = .1692129E-07  DELTAX=  -.2503333E-04
K= 3  X=    3.0000130  Y = .4311230E-08  DELTAX=  -.1251682E-04
K= 4  X=    3.0000060  Y = .1118856E-08  DELTAX=  -.6318053E-05
K= 5  X=    3.0000030  Y = .3008164E-09  DELTAX=  -.3218641E-05
K= 6  X=    3.0000020  Y = .7520397E-10  DELTAX=  -.1668927E-05
K= 7  X=    3.0000010  Y = .2455638E-10  DELTAX=  -.8344643E-06
K= 8  X=    3.0000000  Y = .6139092E-11  DELTAX=  -.4768370E-06
K= 9  X=    3.0000000  Y = .1534773E-11  DELTAX=  -.2384185E-06

X=    3.0000000 APPROXIMATES REQUIRED ROOT TO 7 SIGNIFICANT DIGITS

ENTER NUMSIG,MAXIT,XO(INITIAL GUESS) 7 10 3.06

K= 0  XO=   3.0600000 YO= .1031492E+00
K= 1  X=    3.0308570 Y = .2650978E-01 DELTAX=  -.2914283E-01
K= 2  X =   3.0156610 Y = .6726048E-02 DELTAX=  -.1519647E-01
K= 3  X =   3.0078910 Y = .1694406E-02 DELTAX=  -.7769739E-02
K= 4  X =   3.0039610 Y = .4252645E-03 DELTAX=  -.3929887E-02
K= 5  X =   3.0019840 Y = .1065284E-03 DELTAX=  -.1976515E-02
K= 6  X =   3.0009930 Y = .2666329E-04 DELTAX=  -.9911961E-03
K= 7  X =   3.0004970 Y = .6668914E-05 DELTAX=  -.4963794E-03
K= 8  X =   3.0002480 Y = .1666815E-05 DELTAX=  -.2483705E-03
K= 9  X =   3.0001240 Y = .4166519E-06 DELTAX=  -.1242007E-03
K=10  X =   3.0000620 Y = .1045567E-06 DELTAX=  -.6210418E-04
DESIRED ACCURACY IS NOT EVIDENT IN 10 ITERATIONS
```

k	X_k	$\Delta X_k=-y_k/f'(X_k)$	$\Delta X_k/\Delta X_{k-1}$	$X_{k+1}=X_k+\Delta X_k$
0	3.06	-0.02914283		3.0308570
1	3.03086	-0.01519647	0.53	3.0156610
2	3.01566	-0.00776974	0.52	3.0078910
3	3.00789	-0.00392989	0.51	3.0039610
4	3.00396		≈ 0.5 ---> linear convergence	

(b) จากการใช้ซับรูทีน SECANT ในหัวข้อ 2.1I ได้ผลดังนี้

```
ENTER DESIRED # SIGNIFICANT DIGITS, IOUT (0-2)UNIT 5? con
7 2

ENTER TWO INITIAL GUESSES (SECOND CLOSER IF POSSIBLE)3.1 3.06

SECANT METHOD     X(0)=SECOND INITIAL GUESS
K       X=XK          Y=F(X)          DELTA-X(K-1)
-1  3.1000000      .2979088E+00
 0  3.0600000      .1031151E+00
 1  3.0388260      .4232788E-01     -.2117419E-01
 2  3.0240810      .1666958E-01
 3  3.0150670      .6193161E-02     -.9014439E-02
 4  3.0094090      .2412796E-02     -.5658159E-02
 5  3.0057980      .9250641E-03     -.3611281E-02
 6  3.0035520      .3204346E-03     -.2245504E-02
 7  3.0023620      .1564026E-03     -.1190005E-02
 8  3.0012280      .1716614E-04     -.1134601E-02
 9  3.0010880      .3433228E-04     -.1398864E-03
10  3.0013680      .6103516E-04      .2799034E-03
11  3.0007280     -.1144409E-04     -.6397792E-03
12  3.0008290      .3814697E-05      .1010016E-03
13  3.0008040      .7629395E-05     -.2527237E-04
14  3.0008540      .1525879E-04      .5054474E-04
15  3.0007530      .0000000E+00     -.1010895E-03

COMPUTED F(    3.0007530) = ZERO.  ITERATION DISCONTINUEDStop - Program
```

k	X_k	$\Delta X_k=(-y_k\Delta X_{k-1})/(y_k-y_{k-1})$	$X_{k+1}=X_k+\Delta X_k$
-1	3.1		
0	3.06	-0.02117419	3.0388260
1	3.03683	-0.01474420	3.0240810
2	3.02409	-0.00901444	3.0150670
3	3.01507	-0.00565816	3.0094090
4	3.00941		

2.3* Perform two iterations of Bairstow's method as indicated.

(a) $p(x)=x^4-5x^3+8.5x^2-6x+2$, $r_o=1$, $s_o=0$.

(b) $p(x)=2x^4+5x^3-5x^2-5x+3$, $r_o=-3$, $s_o=-3$.

Given r_o =-2.5 and s_o =1.5 find Q(x) then roots of p(x) = 0.

(c) $p(x)=x^5+x^4-14.8x^3+23.4x^2-12.6x+2$, $r_o=2$, $s_o=-2$.

(d) $p(x)=x^4-x^3+x^2+2$, $r_o=2$, $s_o=-1$.

Given r, =2 and s_7 =-2 find Q(x) then roots of p(x) = 0.

(a) จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X))A(1),...,A(N+1)UNIT 5? con
4 1 -5 6.5 -6 2

ENTER DESIRED # OF SIGNIFICANT DIGITS 7

ENTER INITIAL R,INITIAL S 1 0
```

k	dr	as	r	s
0	-		1.0000000	.0000000
1	.3333333E+00	-.3333333E+00	1.3333330	-.3333333
2	-.2457290E+01	.9171910E-03	-1.1239570	-.3324161
3	.1623954E+01	.2242086E+01	.4999977	1.9096700
4	.4111383E+01	.5665622E+01	4.6113810	7.5152920
5	-.1475153E+01	.9587331E+00	3.1362270	8.5340250
6	-.3111462E+00	-.4563456E+01	2.8250810	3.3705690
7	-.1295367E+00	-.2560515E+01	2.6955440	1.4100540
8	-.2755806E-01	-.1405497E+01	2.6679860	.0045568
9	.6794561E-01	-.8813441E+00	2.7359320	-.8767874
10	.3254108E+00	-.1047277E+01	3.0613430	-1.3240640
11	.6855863E+02	-.1312577E+03	71.6199700	-133.1817000
12	-.6779867E+02	.4726202E+04	3.8213040	4533.0210000
13	-.6608623E+00	-.2295763E+04	3.1604420	2297.2580000
14	-.3302021E+00	-.1148864E+04	2.8302400	1148.3330000
15	-.1649725E+00	-.5746778E+03	2.6652670	573.7156000
16	-.8231159E-01	-.2874006E+03	2.5823560	266.3150000
17	-.4082663E-01	-.1437162E+03	2.5421290	142.5386000
18	-.1976046E-01	-.7186325E+02	2.5223630	70.7355300
19	-.8575789E-02	-.3593514E+02	2.5137330	34.8003900
20	-.1678586E-02	-.1797293E+02	2.5121150	16.8274700

```
Q(X)=X*X-( 2.5121150)*X-( 16.8274700)
COEFFICIENTS OF Q(X):
      1.0000000 -2.4862070 37.0505800Stop - Program terminated.
```

(b) จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)UNIT 5? con
4 2 5 -5 -5 3

ENTER DESIRED # OF SIGNIFICANT DIGITS 7

ENTER INITIAL R,INITIAL S -3 -3

  k       dr            ds             r           s
  0                                  -3.0000000  -3.0000000
  1  .5862069E+00 .3729064E+01 -2.4137930    .7290640
  2  -.7042368E+00  -.3164465E+00  -3.1180300    .4126175
  3   .4281916E+00   .7604384E+00  -2.6898380   1.1730560
  4   .1644644E+00 .2797683E+00  -2.5253140   1.4526240
  5 .2482908E-01.4603647E-01 -2.5005450   1.4988610
  6   .5443016E-03  .1138832E-02 -2.5000000   1.4999990
  1   .2384186E-06   .5960467E-06   -2.5000000    1.5000000
  8.0000000E+00.0000000E+00 -2.5000000    1.5000000
q(X)=X*X- ( -2.5000000)*X- (  1.5000000)
COEFFICIENTS O F  Q(X):
      2.0000000  .0000000 -2.0000000Stop - Program terminated.
```

$$p(x) = q(x)Q(x)$$

$$= (x^2 + 2.5x - 1.5)(2x^2 - 2) = 0$$

ดังนั้นรากทั้ง 4 ของ p(x) คือ -3, 0.5, -1, 1

(c) จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)UNIT 5? con
5 1 1 -14.8 23.4 -12.6 2

ENTER DESIRED # OF SIGNIFICANT DIGITS7

ENTER INITIAL R,INITIAL S 2 -2

  k       dr           as             r            s
  0        -                        2 .0000000 - 2 .0000000
  1  -.6900719E-01   .7022645E+00 1.9309930 -1.2917350
  2  -.8598252E-01   .3603986E+00 1.8450100 -.9373369
  3  -.8397379E-01   .1673752E+00  1 .7610370  -.7699617
  4  -.3504417E-01   .4438463E-01  1  .7259920  -.7255771
  5  -.2386683E-02   .1976986E-02 1.7236060 -.7236001
  6   .1131896E-05  -.6814530E-05 1.7236070 -.7236069
  7  -.2698123E-06   .1325124E-06 1.7236060 -.7236068
  a   .3631497E-06  -.1597914E-06   1.7236070     -.7236069
  9  -.2155426E-06   .1059112E-06 1.7236070 -.7236068
 10   .1645017E-07  -.1087836E-07 1.7236070 -.7236068
q(X)=X*X- ( 1.7236070)*X- (  -.7236068)
COEFFICIENTS OF Q(X):
      1.0000000   2.7236070 -10.6291800 2.7639320Stop - Program terminated.
```

(d) จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)UNIT 5? con
4 1 -1 1 0 2

ENTER DESIRED # OF SIGNIFICANT DIGITS7

ENTER INITIAL R,INITIAL S2 -1

  k       dr           ds             r           s
  0        -                       2.0000000   -1 .0000000
  1  -.4285714E+00   .0000000E+00 1.5714290   -1.0000000
  2   .9950410E+00  -.1908089E+01 2.5654690   -2.9080890
  3  -.4291627E+00   .8093784E+00 2.1373070   -2.0987110
  4  -.1388755E+00   .1237867E+00 1.9984310   -1.9749240
  5   .1479502E-02  -.2494785E-01 1.9999110   -1.9998720
  E   .8929541E-04  -.1277941E-03  2 .0000000  -2.0000000
  7   .0000000E+00   .0000000E+00 2.0000000   -2.0000600
q(X)=X*X- ( 2.0000000)*X- ( -2.00000001
COEFFICIENTS O F Q(X):
      1.0000000   1.00000013 1.0000000Stop - Program terminated.
```

$p(x) = q(x))Q(x) = (x^2 - 2x + 2)(x^2 + x + 1) = 0$

ดังนั้นรากทั้ง 4 ของ $p(x)$ คือ $1+i$, $1-i$, $(-1+\sqrt{3}\,i)/2$, $(-1-\sqrt{3}\,i)/2$

2.4 The polynomial $p(x)=x^4-10x^3+35x^2-50x+24$ has roots

$\bar{x}$ = 1, 2, 3, 4

(a)* Starting with r_o = 3.2, s_o = -2.2 [to find $q(x)=(x-1)(x-2)$] use Bairstow's method to find r_k and s_k until successive iterates agree to 3s. Deflate $p(x)$ and find the error in the roots of the deflated $p(x)$ (whose roots, if exact would be 3 and 4).

จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)UNIT 5? con
4 1 -10 35 -50 24

ENTER DESIRED # OF SIGNIFICANT DIGITS 6

ENTER INITIAL R,INITIAL S 3.2 -2.2

  k        dr             ds
  0                                  3.2~00000    -2.20800000
  1   -.2576334E+00   .2717933E+00   2.9423670    -1.9282070
  2    .5642680E-01  -.6837445E-01   2.9987930    -1.9965810
  3    .1211071E-02  -.3418483E-02   3.0000050    -2.0000000
  4   -.5006804E-05  -.3164127E-11   3. 0000000   -2.0000000
5     -.3973627E-06   .3178899E-06   2.9999990    -1.9999990
q(X)=X*X- (   2.9999990)*X- ( -1.9999990)
COEFFICIENTS OF Q(X):
       1.0000000   -7.0000000 12.0000000Stop - Program terminated.
```

(b)* Starting with r_o = 7.3, s_o = -12.3 [to find $q(x) = (x-3)(x-4)$] use Bairstow's method to find r_k and s_k until successive iterates agree to 3s. Deflate $p(x)$ and find the error in the roots of the deflated $p(x)$ (whose roots, if exact would be 1 and 2).

จากการใช้ซับรูทีน BAIRSTOW ในภาคผนวก ค (บทที่ 2) ได้ผลดังนี้

```
ENTER N(DEGREE OF p(X)) A(1),...,A(N+1)UNIT 5? con
4 1 -10 35 -50 24

ENTER DESIRED # OF SIGNIFICANT DIGITS 7

ENTER INITIAL, R,INITIAL S 7.3 -12.3

  k        dr             ds
  0                                      7.3~00000   -12.3~00000
  1  -.1066013E+01   .4529377E+01   6.2339870    -7.7706230
  2   .4197915E+00  -.3009149E+01   6.6537790   -10.7797700
  3   .3365701E+00  -.1063662E+01   6.9903490   -11.8434300
  4   .6353718E-02  -.1420759E+00   6.9967030   -11.9855100
  5   .3312822E-02  -.1455611E-01   7.0000160   -12.0000700
  6  -.1573581E-04   .6580497E-04   7.0000000   -12.0000000
  7   .0000000E+00   .0000000E+00   7.0000000   -12.0000000
q(X)=X*X- (   7.0000000)*X- ( -12.0000000)
COEFFICIENTS  OF  Q(X):
       1.0000000  -3.0000000  2.0000000Stop - Program  terminated.
```