ภาคผนวก 3
สูตรการอินทิเกรต

1. $\int u\,dv = uv - \int v\,du$

2. $\int a^u du = \frac{a^u}{\ln a} + C,\ a \neq 1,\ a > 0$

3. $\int \cos u\,du = \sin u + C$

4. $\int \sin u\,du = -\cos u + C$

5. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C,\ n \neq -1$

6. $\int (ax+b)^{-1} dx = \frac{1}{a}\ln|ax+b| + C$

7. $\int x(ax+b)^n dx = \frac{(ax+b)^{n+1}}{a^2}\left(\frac{ax+b}{n+2} - \frac{b}{n+1}\right] + C,\ n \neq -1, -2$

8. $\int x(ax+b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$

9. $\int x(ax+b)^{-2} dx = \frac{1}{a^2}\left[\ln|ax+b| + \frac{b}{ax+b}\right] + C$

10. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C$

11. $\int (\sqrt{ax+b})^n\,dx = \frac{2}{a}\frac{(\sqrt{ax+b})^{n+2}}{n+2} + C,\ n \neq -2$

12. $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$

13. (a) $\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}} + C,$ if $b < 0$

    (b) $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C,$ if $b > 0$

14. $\int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}} + C$

15. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} + C$

16. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

17. $\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\tan^{-1}\frac{x}{a} + C$

18. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$

19. $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{2a^2}\int \frac{dx}{a^2-x^2}$

20. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sin h^{-1}\frac{x}{a} + C = \ln|x+\sqrt{a^2+x^2}| + C$

21. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sin h^{-1}\frac{x}{a} + C$

22. $\int x^2\sqrt{a^2+x^2}\,dx = \frac{x(a^2+2x^2)\sqrt{a^2+x^2}}{8} - \frac{a^4}{8}\sin h^{-1}\frac{x}{a} + C$

23. $\int \frac{\sqrt{a^2+x^2}}{x}dx = \sqrt{a^2+x^2} - a\sin h^{-1}\left|\frac{a}{x}\right| + C$

24. $\int \frac{\sqrt{a^2+x^2}}{x^2}dx = \sin h^{-1}\frac{x}{a} - \frac{\sqrt{a^2+x^2}}{x} + C$

25. $\int \frac{x^2}{\sqrt{a^2+x^2}}dx = -\frac{a^2}{2}\sin h^{-1}\frac{x}{a} + \frac{x\sqrt{a^2+x^2}}{2} + C$

26. $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

27. $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$

28. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

29. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

30. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{a^4}{8}\sin^{-1}\frac{x}{a} - \frac{1}{8}x\sqrt{a^2-x^2}\,(a^2-2x^2) + C$

31. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

32. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\sin^{-1}\frac{x}{a} - \frac{\sqrt{a^2-x^2}}{x} + C$

33. $\int \frac{x^2}{\sqrt{a^2-x^2}}dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + C$

34. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

35. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

36. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos h^{-1}\frac{x}{a} + C = \ln|x+\sqrt{x^2-a^2}| + C$

37. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cos h^{-1}\frac{x}{a} + C$

38. $\int (\sqrt{x^2-a^2})^n dx = \frac{x(\sqrt{x^2-a^2})^n}{n+1} - \frac{na^2}{n+1}\int (\sqrt{x^2-a^2})^{n-2}dx,\ n \neq -1$

39. $\int \frac{dx}{(\sqrt{x^2-a^2})^n} = \frac{x(\sqrt{x^2-a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2}\int \frac{dx}{(\sqrt{x^2-a^2})^{n-2}},\ n \neq 2$

40. $\int x(\sqrt{x^2-a^2})^n dx = \frac{(\sqrt{x^2-a^2})^{n+2}}{n+2} + C,\ n \neq -2$

41. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\cos h^{-1}\frac{x}{a} + C$

42. $\int \frac{(\sqrt{x^2-a^2})}{x}dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right| + C$

43. $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = \cos h^{-1}\frac{x}{a} - \frac{\sqrt{x^2-a^2}}{x} + C$

44. $\int \frac{x^2}{\sqrt{x^2-a^2}}dx = \frac{a^2}{2}\cos h^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{x^2-a^2} + C$

45. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C$

46. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$

47. $\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right) + C$

48. $\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

49. $\int(\sqrt{2ax-x^2})^n dx = \frac{(x-a)(\sqrt{2ax-x^2})^n}{n+1} + \frac{na^2}{n+1}\int(\sqrt{2ax-x^2})^{n-2}dx$

50. $\int\frac{dx}{(\sqrt{2ax-x^2})^n} = \frac{(x-a)(\sqrt{2ax-x^2})^{2-n}}{(n-2)a^2} + \frac{(n-3)}{(n-2)a^2}\int\frac{dx}{(\sqrt{2ax-x^2})^{n-2}}$

51. $\int x\sqrt{2ax-x^2}\,dx = \frac{(x+a)(2x-3a)\sqrt{2ax-x^2}}{6} + \frac{a^3}{2}\sin^{-1}\frac{x-a}{a} + C$

52. $\int\frac{\sqrt{2ax-x^2}}{x}dx = \sqrt{2}ax-x^2 + a\,\sin^{-1}\frac{x-a}{a} + C$

53. $\int\frac{\sqrt{2ax-x^2}}{x^2}dx = -2\sqrt{\frac{2a-x}{x}} - \sin^{-1}\left(\frac{x-a}{a}\right) + C$

54. $\int\frac{x\,dx}{\sqrt{2ax-x^2}} = a\,\sin^{-1}\frac{x-a}{a} - \sqrt{2ax-x^2} + C$

55. $\int\frac{dx}{x\sqrt{2ax-x^2}} = -\frac{1}{a}\sqrt{\frac{2a-x}{x}} + C$

56. $\int\sin ax\,dx = -\frac{1}{a}\cos ax + C$

57. $\int\cos ax\,dx = \frac{1}{a}\sin ax + C$

58. $\int\sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$

59. $\int\cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$

60. $\int\sin^n ax\,dx = \frac{-\sin^{n-1}ax\cos ax}{na} + \frac{n-1}{n}\int\sin^{n-2}ax\,dx$

61. $\int\cos^n ax\,dx = \frac{\cos^{n-1}ax\sin ax}{na} + \frac{n-1}{n}\int\cos^{n-2}ax\,dx$

62. (a) $\int\sin ax\cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C,\ a^2 \neq b^2$

(b) $\int\sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)},\ a^2 \neq b^2$

(c) $\int\cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)},\ a^2 \neq b^2$

63. $\int \sin ax \cos ax\, dx = -\frac{\cos 2ax}{4a} + C$

64. $\int \sin^n ax \cos ax\, dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

65. $\int \frac{\cos ax}{\sin ax} dx = \frac{1}{a} \ln|\sin ax| + C$

66. $\int \cos^n ax \sin ax\, dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

67. $\int \frac{\sin ax}{\cos ax} dx = -\frac{1}{a} \ln|\cos ax| + C$

68. $\int \sin^n ax \cos^m ax\, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax\, dx,$

$n \neq -m$ (If $n = -m$ use No. 86.)

69. $\int \sin^n ax \cos^m ax\, dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax\, dx,$

$m \neq -n$ (If $m = -n$, use No. 87.)

70. $\int \frac{dx}{b + c \sin ax} = \frac{-2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\frac{b-c}{b+c}} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right] + C, \quad b^2 > c^2$

71. $\int \frac{dx}{b + c \sin ax} = \frac{-1}{a\sqrt{c^2 - b^2}} \ln\left|\frac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax}\right| + C, \quad b^2 < c^2 -$

72. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

73. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a} \tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

74. $\int \frac{dx}{b + c \cos ax} = \frac{2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2}\right] + C, \quad b^2 > c^2$

75. $\int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \ln\left|\frac{c + b \cos ax + \sqrt{c^2 - b^2} \sin ax}{b + c \cos as}\right| + C, \quad b^2 < c^2$

76. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$

77. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$

78. $\int s \sin ax\, dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$

79. $\int x \cos ax\, dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$

80. $\int x^n \sin ax\, dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx$

81. $\int x^n \cos ax\, dx = \frac{x^n}{a}\sin ax - \frac{n}{a}\int x^{n-1}\sin ax\, dx$

82. $\int \tan ax\, dx = -\frac{1}{a}\ln|\cos ax| + C$

83. $\int \cot ax\, dx = \frac{1}{a}\ln|\sin ax| + C$

84. $\int \tan^2 ax\, dx = \frac{1}{a}\tan ax - x + C$

85. $\int \cot^2 ax\, dx = -\frac{1}{a}\cot ax - x + C$

86. $\int \tan^n ax\, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax\, dx,\ n \neq 1$

87. $\int \cot^n ax\, dx = -\frac{\cot^{n-1} dx}{a(n-1)} - \int \cot^{n-2} ax\, dx,\ n \neq 1$

88. $\int \sec ax\, dx = \frac{1}{a}\ln|\sec ax + \tan ax| + C$

89. $\int \csc ax\, dx = -\frac{1}{a}\ln|\csc ax + \cot ax| + C$

90. $\int \sec^2 ax\, dx = \frac{1}{a}\tan ax + C$

91. $\int \csc^2 ax\, dx = -\frac{1}{a}\cot ax + C$

92. $\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} ax\, dx,\ n \neq 1$

93. $\int \csc^n ax\, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} ax\, dx,\ n \neq 1$

94. $\int \sec^n ax \tan ax\, dx = \frac{\sec^n ax}{na} + C,\ n \neq 0$

95. $\int \csc^n ax \cot ax\, dx = -\frac{\csc^n ax}{na} + C,\ n \neq 0$

96. $\int \sin^{-1} ax\, dx = x \sin^{-1} ax + \frac{1}{a}\sqrt{1 - a^2x^2} + C$

97. $\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \frac{1}{a}\sqrt{1 - a^2x^2} + C$

98. $\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \frac{1}{2a}\ln(1 + a^2x^2) + C$

99. $\int x^n \sin^{-1} ax\, dx = \frac{x^{n+1}}{n+1}\sin^{-1} ax - \frac{a}{n+1}\int \frac{x^{n+1}dx}{\sqrt{1 - a^2x^2}},\ n \neq -1$

100. $\int x^n \cos^{-1} ax\, dx = \frac{x^{n+1}}{n+1}\cos^{-1} ax + \frac{a}{n+1}\int \frac{x^{n+1}dx}{\sqrt{1 - a^2x^2}},\ n \neq -1$

101. $\int x^n \tan^{-1} ax\, dx = \frac{x^{n+1}}{n+1}\tan^{-1} ax - \frac{a}{n+1}\int \frac{x^{n+1}dx}{\sqrt{1 - a^2x^2}},\ n \neq -1$

102. $\int e^{ax} dx = \frac{1}{a}e^{ax} + C$

103. $\int b^{ax} dx = \frac{1}{a}\frac{b^{ax}}{\ln b} + C,\ b > 0,\ b \neq 1$

104. $\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + C$

105. $\int x^n e^{ax} dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1} e^{ax} dx$

106. $\int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b}\int x^{n-1} b^{ax} dx,\ b > 0,\ b \neq 1$

107. $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + C$

108. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$

109. $\int \ln ax\, dx = x \ln ax - x + C$

110. $\int x^n \ln ax\, dx = \frac{x^{n+1}}{n+1}\ln ax - \frac{x^{n+1}}{(n+1)^2} + C,\ n \neq 1$

111. $\int x^{-1} \ln ax\, dx = \frac{1}{2}(\ln ax)^2 + C$

112. $\int \frac{dx}{x \ln ax} = \ln|\ln ax| + C$

113. $\int \sin h\, ax\, dx = \frac{1}{a}\cos h\, ax + C$

114. $\int \cos h\, ax\, dx = \frac{1}{a}\sin h\, ax + C$

115. $\int \sin h^2 ax\, dx = \frac{\sin h\, 2ax}{4a} - \frac{x}{2} + C$

116. $\int \cos h^2 ax\, dx = \frac{\sin h\, 2ax}{4a} + \frac{x}{2} + C$

117. $\int \sin h^n ax\, dx = \frac{\sin h^{n-1} ax \cos h\, ax}{na} - \frac{n-1}{n}\int \sin h^{n-2} ax\, dx,\ n \neq 0$

118. $\int \cos h^n ax\, dx = \frac{\cos h^{n-1} ax \sin h\, ax}{na} + \frac{n-1}{n}\int \cos h^{n-2} ax\, dx,\ n \neq 0$

119. $\int x \sin h\, ax\, dx = \frac{x}{a}\cos h\, ax - \frac{1}{a^2}\sin h\, ax + C$

120. $\int x \cos h\, ax\, dx = \frac{x}{a}\sin h\, ax - \frac{1}{a^2}\cos h\, ax + C$

121. $\int x^n \sin h\, ax\, dx = \frac{x^n}{a}\cos h\, ax - \frac{n}{a}\int x^{n-1}\cos h\, ax\, dx$

122. $\int x^n \cos h\, ax\, dx = \frac{x^n}{a}\sin h\, ax - \frac{n}{a}\int x^{n-1}\sin h\, ax\, dx$

123. $\int \tan h\, ax\, dx = \frac{1}{a}\ln(\cos h\, ax) + C$

124. $\int \cot h\, ax\, dx = \frac{1}{a}\ln|\sin h\, ax| + C$

125. $\int \tan h^2 ax\, dx = x - \frac{1}{a}\tan h\, ax + C$

126. $\int \cot h^2 ax\, dx = x - \frac{1}{a}\cot h\, ax + C$

127. $\int \tan h^n ax\, dx = -\frac{\tan h^{n-1} ax}{(n-1)a} + \int \tan h^{n-2} ax dx,\ n \neq 1$

128. $\int \cot h^n ax\, dx = -\frac{\cot h^{n-1} ax}{(n-1)a} + \int \cot h^{n-2} ax\, dx,\ n \neq 1$

129. $\int \sec h\, ax\, dx = \frac{1}{a}\sin^{-1}(\tan h\, ax) + C$

130. $\int \csc h\, ax\, dx = \frac{1}{a}\ln\left|\tan h \frac{ax}{2}\right| + C$

131. $\int \sec h^2 ax\, dx = \frac{1}{a}\tan h\, ax + C$

132. $\int \csc h^2 ax\, dx = -\frac{1}{a}\cot h\, ax + C$

133. $\int \sec h^n ax\, dx = \frac{\sec h^{n-2} ax \tan h\, ax}{(n-1)a} + \frac{n-2}{n-1}\int \sec h^{n-2} ax\, dx,\ n \neq 1$

134. $\int \csc h^n ax\, dx = -\frac{\csc h^{n-2} ax \cot h\, ax}{(n-1)a} - \frac{n-2}{n-1}\int \csc h^{n-2} ax\, dx,\ n \neq 1$

135. $\int \sec h^n ax \tan h\, ax\, dx = -\frac{\sec h^n ax}{na} + C,\ n \neq 0$

136. $\int \csc h^n ax \cot h\, ax\, dx = -\frac{\csc h^n ax}{na} + C,\ n \neq 0$

137. $\int e^{ax} \sin h\, bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b}\right] + C,\ a^2 \neq b^2$

138. $\int e^{ax} \cos h\, bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b}\right] + C,\ a^2 \neq b^2$

139. $\int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n) = (n-1)!\ n > 0$

140. $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}},\ a > 0$

141. $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \begin{cases} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot n} \cdot \frac{\pi}{2}, & \text{if n is an even integer} \geqslant 2, \\ \frac{2\cdot 4\cdot 6\cdot \ldots \cdot (n-1)}{3\cdot 5\cdot 7\cdot \ldots \cdot n}, & \text{if n is an odd integer} \geqslant 3 \end{cases}$