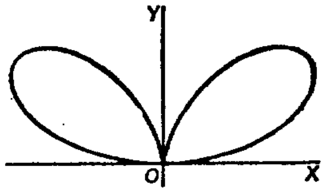
ภาคผนวก

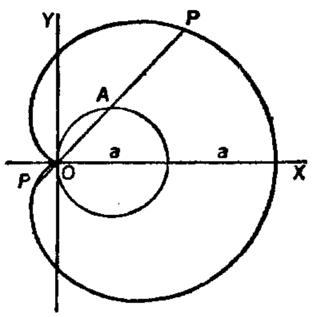
เส้นโค้งเพื่อการอ้างอิง

Bifolium



$$(x^2 + y^2)^2 = ax^2y$$
$$r = a\sin\theta\cos^2\theta$$

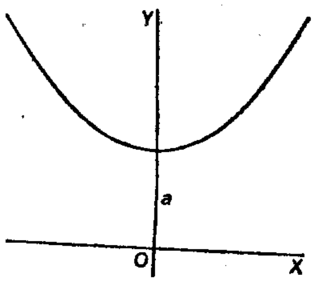
Cardioid



$$(x^2 + y^2 - ax)^2 = a^2(x^2 + y^2)$$
$$r = a(\cos\theta + 1)$$
or
$$r = a(\cos\theta - 1)$$
$$[P'A = AP = a]$$

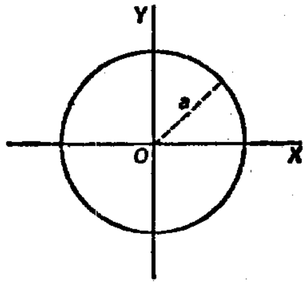
Cassinian curves
See: Ovals of Cassini

Catenary, Hyperbolic cosine



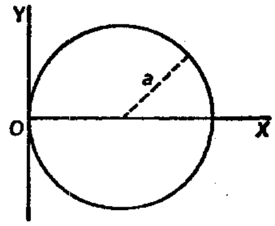
$$y = \frac{x}{a}(e^{x/a} + e^{-x/a}) = a\cosh\frac{x}{a}$$

Circle
(a)



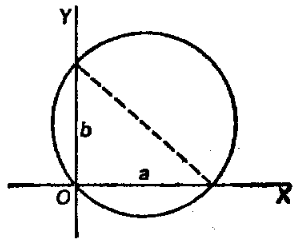
$$x^2 + y^2 = a^2$$
$$r = a$$

(b)



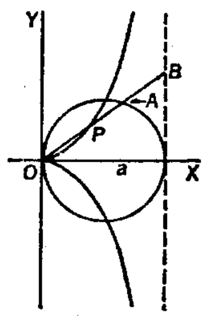
$$x^2 + y^2 = 2ax$$
$$r = 2a\cos\theta$$

(c)



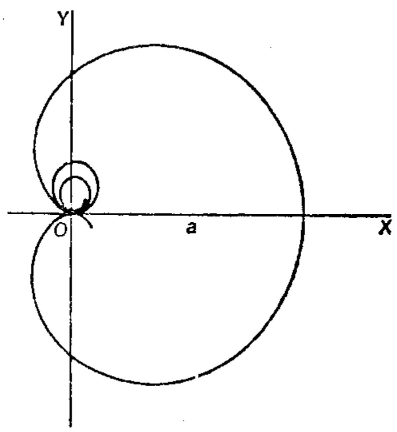
$$x^2 + y^2 = ax + by$$
$$r = a\cos\theta + b\sin\theta$$

Cissoid of Diocles



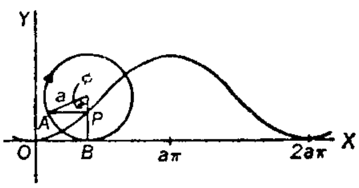
$$y^2(a - x) = x^3$$
$$r = a\sin\theta\tan\theta$$
$$[OP = AB]$$

Cochleoid, Oui-ja board curve



$$(x^2 + y^2)\tan^{-1}(y/x) = ay$$
$$r\theta = a\sin\theta$$

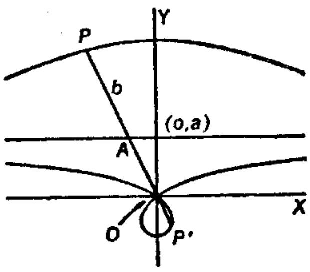
Companion to the cycloid



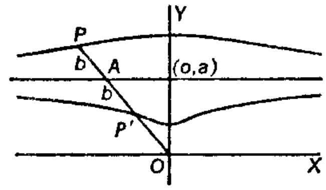
$$\begin{cases} x = a\phi \\ y = a(1 - \cos\phi) \end{cases}$$
$$[OB = \widehat{AB}]$$
(This is a sinusoid)

Conchoid of Nicomedes

(a) $a < b$



(b) $a > b$

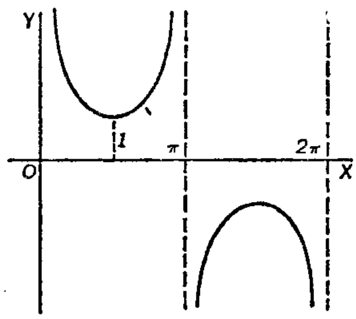


$$(y - a)^2(x^2 + y^2) = b^2y^2$$
$$r = a\csc\theta \pm b$$
$$[P'A = AP = b]$$

Conic sections

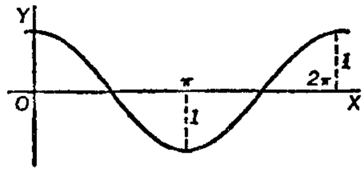
See: Circle; Ellipse; Hyperbola; Parabola

Cosecant curve



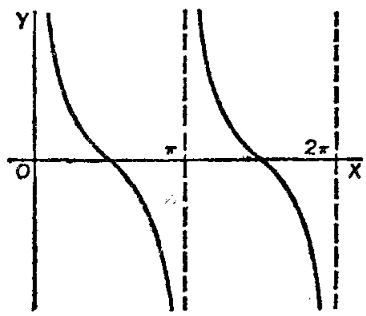
$y = \csc x$

Cosine curve



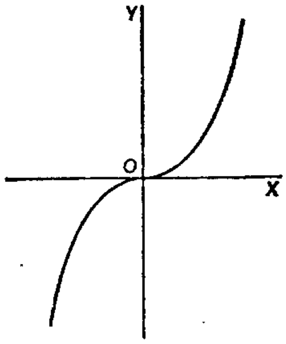
$y = \cos x$

Cotangent curve



$y = \cot x$

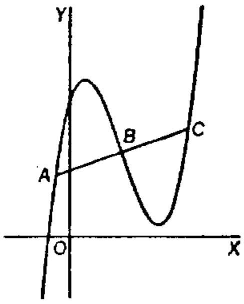
Cubical parabola (special)



$$y = ax^3, \quad a > 0$$

$$r^2 = \frac{1}{a}\sec^2\theta\tan\theta, \quad a > 0$$

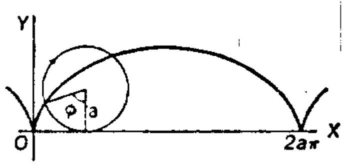
Cubical parabola (general)



$$y = ax^3 + bx^2 + cx + d, \quad a > 0$$
$$[AB = BC]$$
(abscissa of $B = -b/3a$)

Curtate cycloid
See: Cycloid, curtate

Cycloid (cusp at origin)

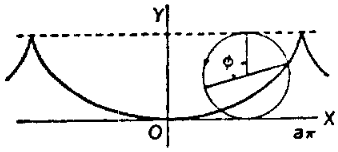


$$x = a \arccos\frac{a-y}{a} \mp \sqrt{2ay - y^2}$$

$$\begin{cases} x = a(\phi - \sin\phi) \\ y = a(1 - \cos\phi) \end{cases}$$

(For one arch: arc length $= 8a$; area $= 3\pi a^2$)

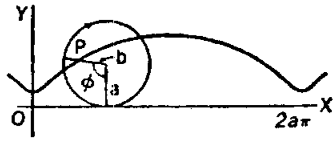
Cycloid (vertex at origin)



$$x = 2a \arcsin\sqrt{y/2a} + \sqrt{2ay - y^2}$$

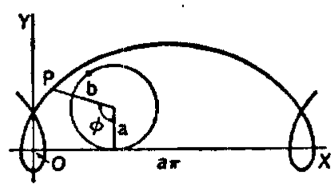
$$\begin{cases} x = a(\phi + \sin\phi) \\ y = a(1 - \cos\phi) \end{cases}$$

Cycloid, curtate



$$\begin{cases} x = a\phi - b\sin\phi \\ y = a - b\cos\phi \end{cases}$$
$$a > b$$

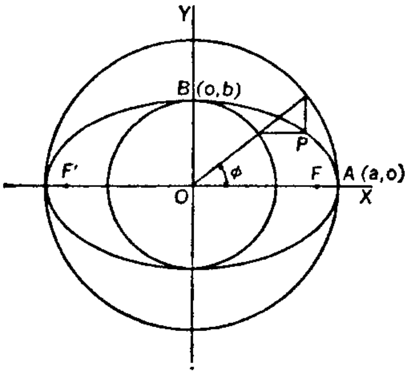
Cycloid, prolate



$$\begin{cases} x = a\phi - b\sin\phi \\ y = a - b\cos\phi \end{cases}$$
$$a < b$$

Deltoid
See: Hypocycloid of three cusps

Ellipse

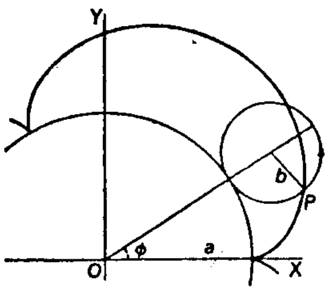


$$x^2/a^2 + y^2/b^2 = 1$$

$$\begin{cases} x = a\cos\phi \\ y = b\sin\phi \end{cases}$$

$$[BF' = BF = a, \quad PF' + PF = 2a]$$

Epicycloid

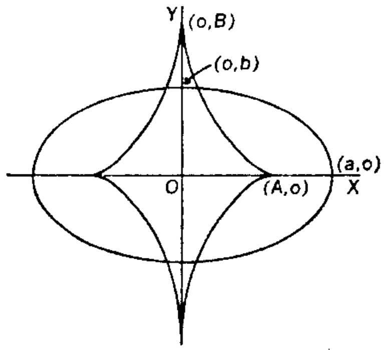


$$\begin{cases} x = (a+b)\cos\phi - b\cos\left(\dfrac{a+b}{b}\phi\right) \\ y = (a+b)\sin\phi - b\sin\left(\dfrac{a+b}{b}\phi\right) \end{cases}$$

Equiangular spiral
See: Spiral, logarithmic or equiangular

Equilateral hyperbola
See: Hyperbola, equilateral or rectangular

Evolute of ellipse

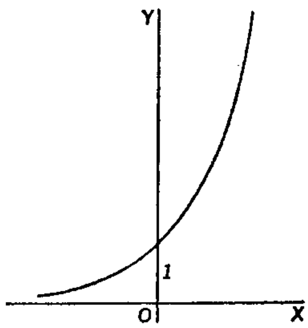


$$(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$$

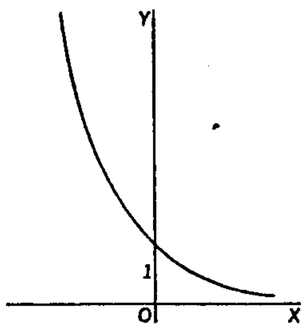
$$\begin{cases} x = A\cos^3\phi \\ y = B\sin^3\phi \end{cases}$$

$$[A = (a^2 - b^2)/a, \quad B = (a^2 - b^2)/b]$$

Exponential curve
(1) $a > 0$

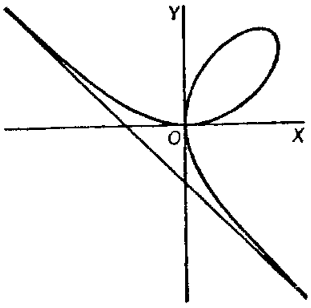


(2) $a < 0$



$$y = e^{ax}$$

Folium of Descartes



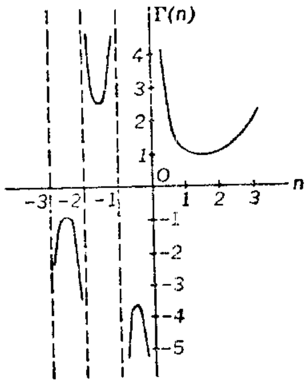
$$x^3 + y^3 - 3axy = 0$$

$$\begin{cases} x = 3a\phi/(1 + \phi^3) \\ y = 3a\phi^2/(1 + \phi^3) \end{cases}$$

$$r = \frac{3a \sin\theta \cos\theta}{\sin^3\theta + \cos^3\theta}$$

[asymptote: $x + y + a = 0$]

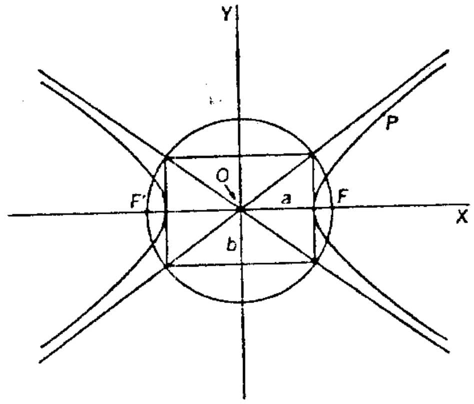
Gamma function



$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx \quad (n > 0)$$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n} \quad (0 > n \neq -1, -2, -3, \ldots)$$

Hyperbola

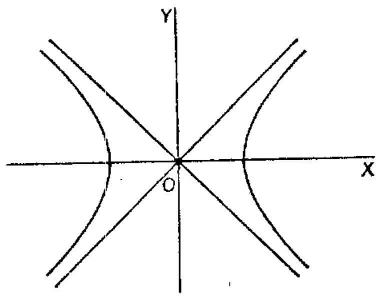


$$x^2/a^2 - y^2/b^2 = 1$$

$$[F'P - FP = 2a]$$

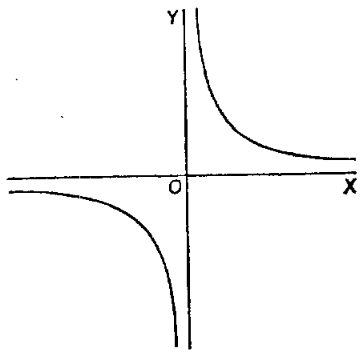
Hyperbola, equilateral or rectangular

(1)



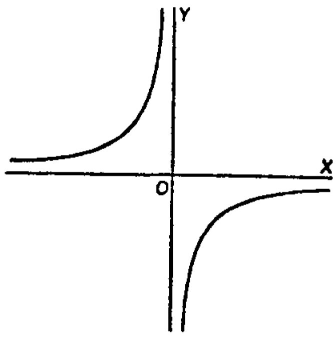
$$x^2 - y^2 = a^2$$

(2)



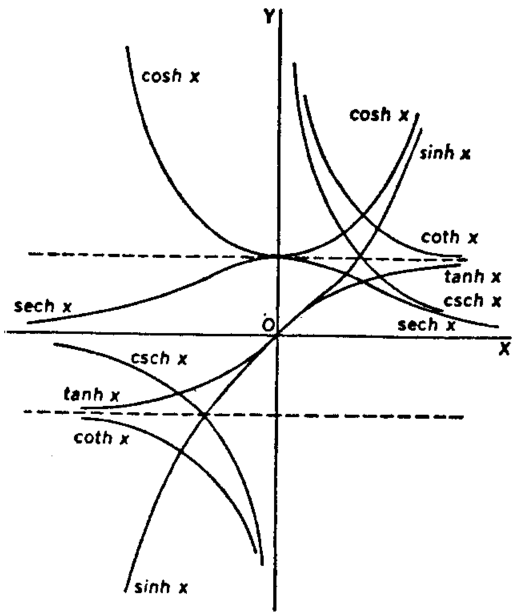
$$xy = k, \quad k > 0$$

(3)



$xy = k, \quad k < 0$

Hyperbolic functions



$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^x - e^{-x}}$$

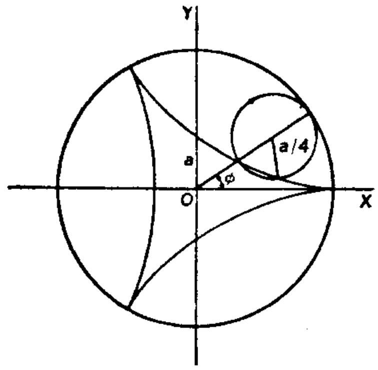
$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Hyperbolic spiral

See: Spiral, hyperbolic or reciprocal

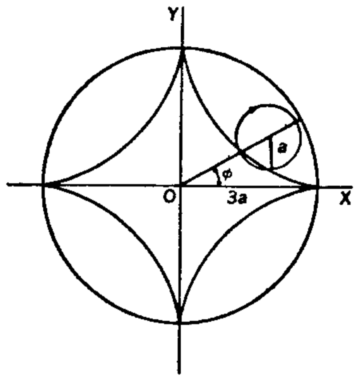
Hypocycloid of three cusps, Deltoid



$$x^{2/3} + y^{2/3} = a^{2/3}$$

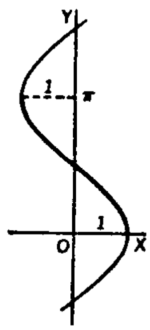
$$\begin{cases} x = a\cos^3\phi \\ y = a\sin^3\phi \end{cases}$$

Hypocycloid of four cusps, Astroid



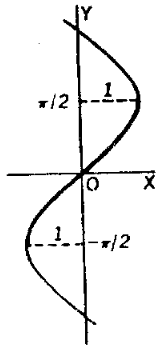
$$\begin{cases} x = 2a\cos\phi + a\cos 2\phi \\ y = 2a\sin\phi - a\sin 2\phi \end{cases}$$

Inverse cosine curve



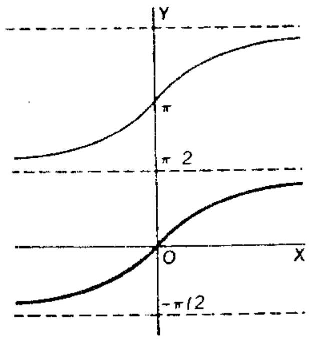
$y = \arccos x$

Inverse sine curve



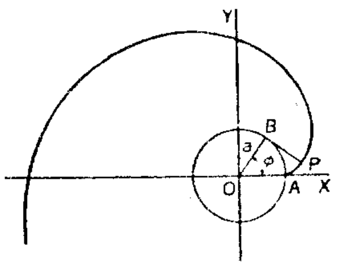
$y = \arcsin x$

Inverse tangent curve



$y = \arctan x$

Involute of circle

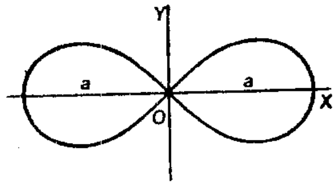


$$\begin{cases} x = a\cos\phi + a\phi\sin\phi \\ y = a\sin\phi - a\phi\cos\phi \end{cases}$$

$[BP = \widehat{BA}]$

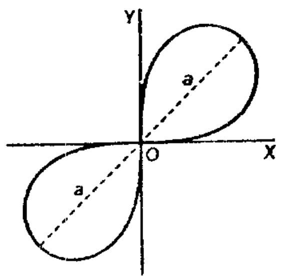
Lemniscate of Bernoulli, Two-leaved rose

(a)



$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$
$$r^2 = a^2\cos 2\theta$$

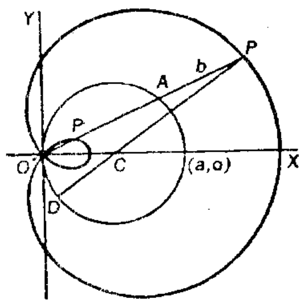
(b)



$$(x^2 + y^2)^2 = 2a^2xy$$
$$r^2 = a^2\sin 2\theta$$

Limaçon of Pascal

(1) $a > b$

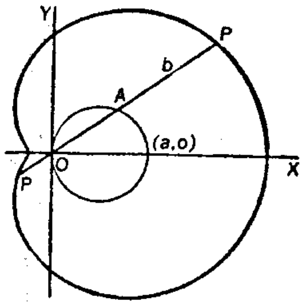


[If $a = 2b$, the curve is called the *trisectrix*, since then $\angle OPD = \frac{1}{3}\angle OCD$.]

(2) $a = b$

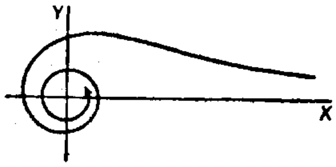
See: Cardioid

(3) $a > b$



$$(x^2 + y^2 - ax)^2 = b^2(x^2 + y^2)$$
$$r = b + a\cos\theta$$
$$[P'A = AP = b]$$

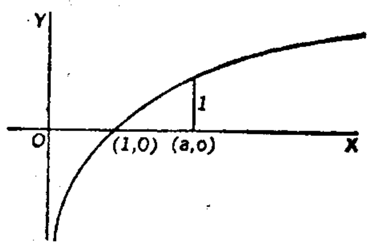
Lituus



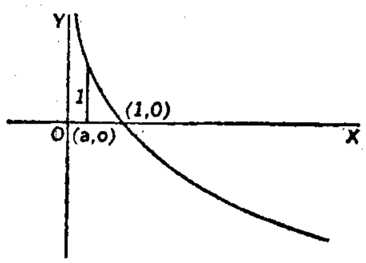
$$r^2\theta = a^2$$

Logarithmic curve

(1) $a > 1$



(2) $0 < a < 1$

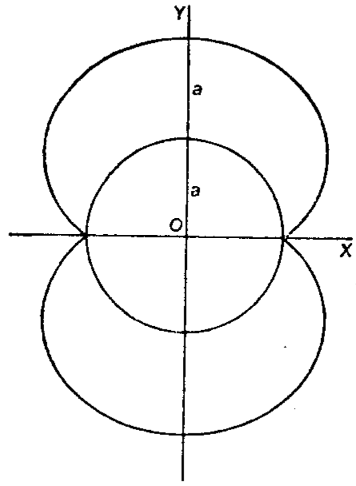


$$y = \log_a x$$

Logarithmic spiral

See: Spiral, logarithmic or equiangular

Nephroid



$$\begin{cases} x = \frac{1}{2}a(3\cos\phi - \cos 3\phi) \\ y = \frac{1}{2}a(3\sin\phi - \sin 3\phi) \end{cases}$$

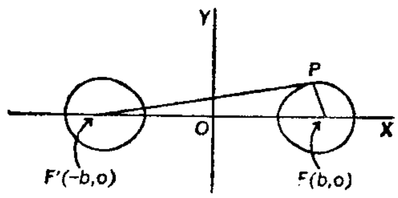
[The nephroid is a 2-cusped epicycloid.]

Oui-ja board curve

See: Cochleoid

Ovals of Cassini

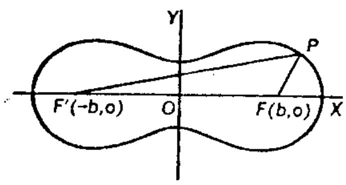
(1) $b > k$



(2) $b = k$

See: Lemniscate of Bernoulli

(3) $b < k$

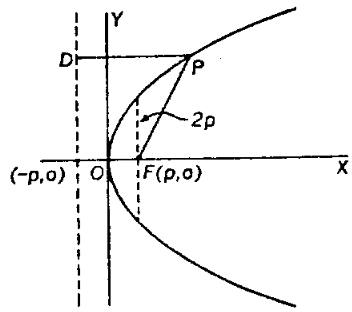


$$(x^2 + y^2 + b^2)^2 - 4b^2x^2 = k^4$$
$$r^4 + b^4 - 2r^2b^2\cos 2\theta = k^4$$
$$[F'P \cdot FP = k^2]$$

[These curves are sections of a torus on planes parallel to the axis of the torus.]

Parabola

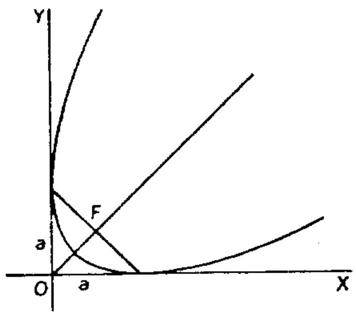
(1)



$y^2 = 4px$

$[DP = FP]$

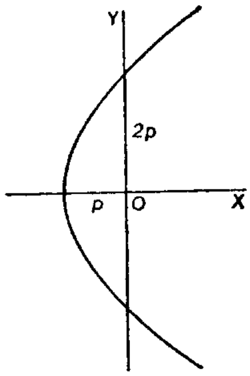
(2)



$\pm x^{1/2} \pm y^{1/2} = a^{1/2}$

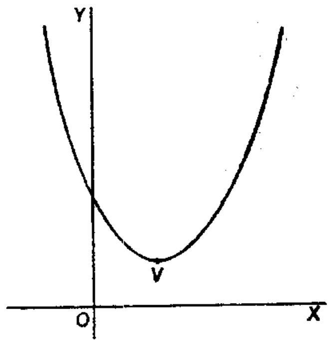
$(x - y)^2 - 2a(x + y) + a^2 = 0$

(3)



$r = 2p/(1 - \cos\theta)$

(4)



$y = ax^2 + bx + c, \quad a > 0$

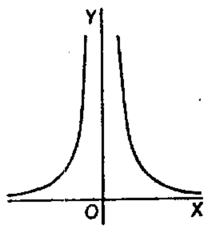
[abscissa of vertex $= -b/2a$]

Parabolic spiral

See: Spiral, parabolic

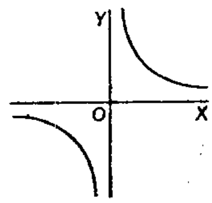
Power functions

(1)



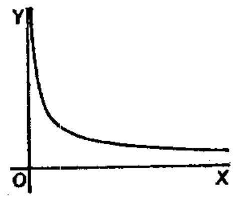
$y = x^{-2}$

(2) Equilateral hyperbola



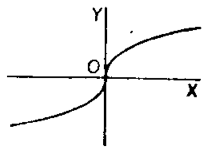
$y = x^{-1}$

(3)



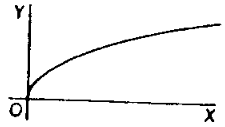
$y = x^{-1/2}$

(4) Cubical parabola



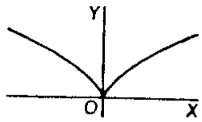
$y = x^{1/3}$

(5) Half of a parabola



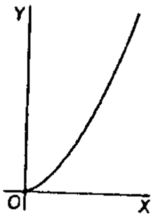
$y = x^{1/2}$

(6) Semicubical parabola



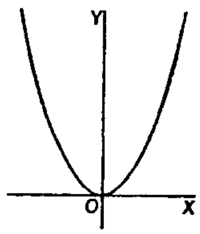
$y = x^{2/3}$

(7) Half of semicubical parabola



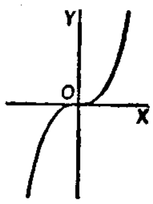
$y = x^{3/2}$

(8) Parabola



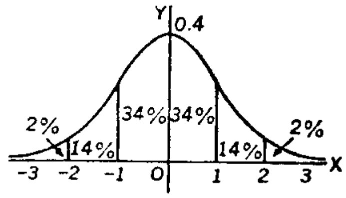
$y = x^2$

(9) Cubical parabola



$y = x^3$

Probability curve

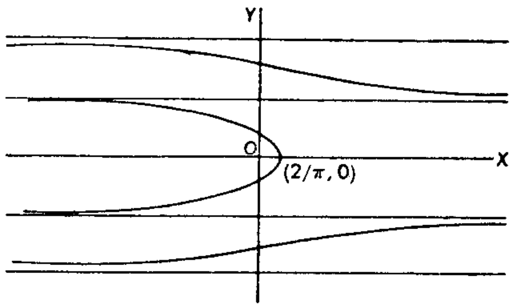


$$y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Prolate cycloid
See: Cycloid, prolate

Pursuit curve
See: Tractrix

Quadratrix of Hippias



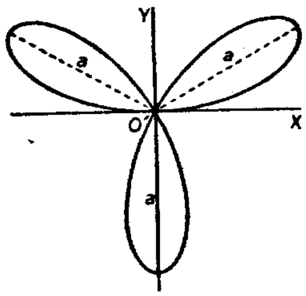
$y = x \tan(\pi y/2)$

Reciprocal spiral
See: Spiral, hyperbolic or reciprocal

Rectangular hyperbola
See: Hyperbola, equilateral or rectangular

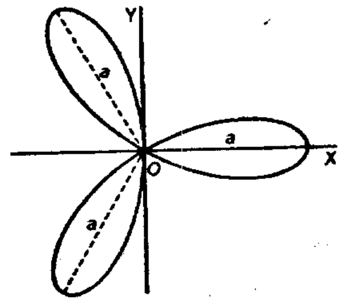
Rose curves
(1) Two-leaved
See: Lemniscate of Bernoulli. Two-leaved rose

(2) Three-leaved



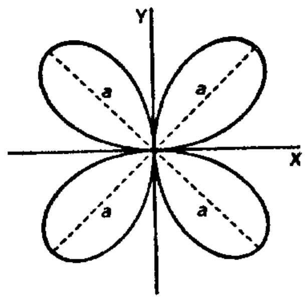
$r = a \sin 3\theta$

(3) Three-leaved



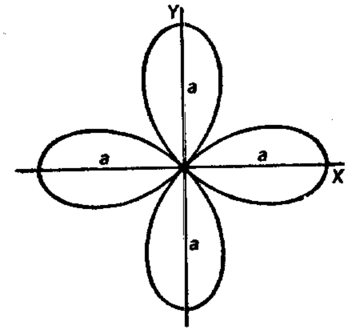
$r = a \cos 3\theta$

(4) Four-leaved



$r = a \sin 2\theta$

(5) Four-leaved

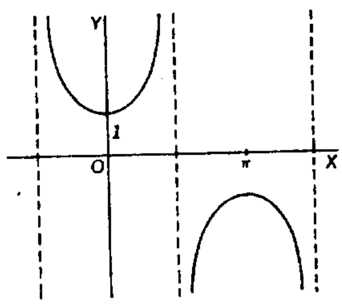


$r = a \cos 2\theta$

(6) n-leaved

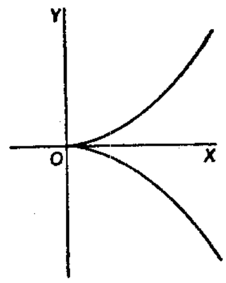
The roses $r = a \sin n\theta$ and $r = a \cos n\theta$, have, for n an even integer, $2n$ leaves; for n an odd integer, n leaves. The roses $r^2 = a \sin n\theta$ and $r^2 = a \cos n\theta$, have, for n an even integer, n leaves; for n an odd integer, $2n$ leaves.

Secant curve



$y = \sec x$

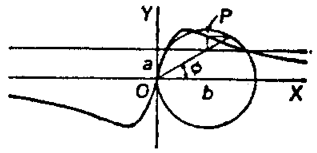
Semicubical parabola



$y^2 = ax^3$

$r = \frac{1}{a} \tan^2 \theta \sec \theta$

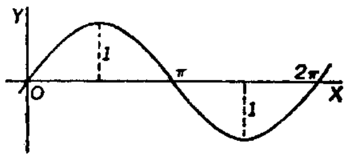
Serpentine curve



$$(a^2 + x^2)y = abx$$

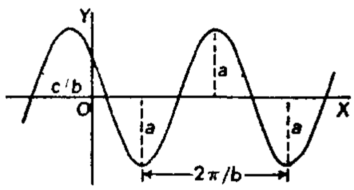
$$\begin{cases} x = a \cot \phi \\ y = b \sin \phi \cos \phi \end{cases}$$

Sine curve



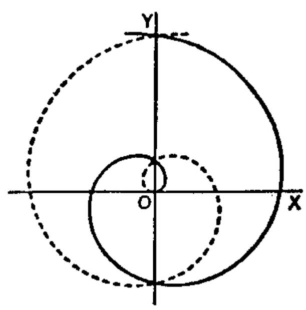
$$y = \sin x$$

Sinusoid



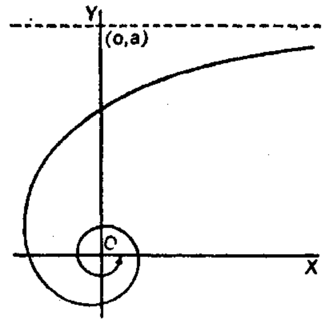
$$y = a \sin (bx + c)$$

Spiral of Archimedes



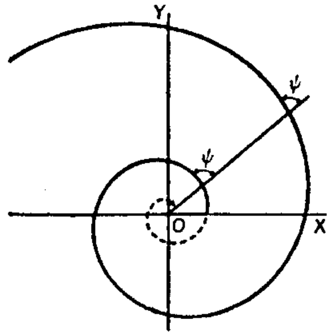
$$r = a\theta$$

Spiral, hyperbolic or reciprocal



$$r\theta = a$$

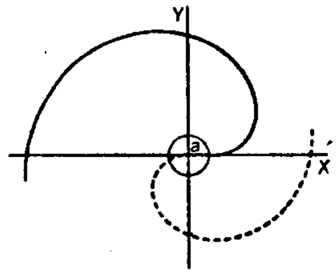
Spiral, logarithmic or equiangular



$$r = e^{a\theta}$$

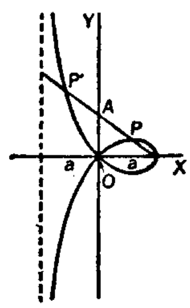
$$\log r = a\theta$$

Spiral, parabolic



$$(r - a)^2 = 4ak\theta$$

Strophoid

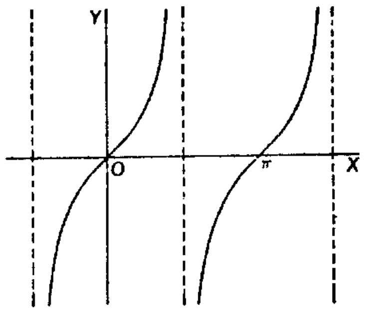


$$y^2 = x^2 \frac{a - x}{a + x}$$

$$r = a \cos 2\theta \sec \theta$$

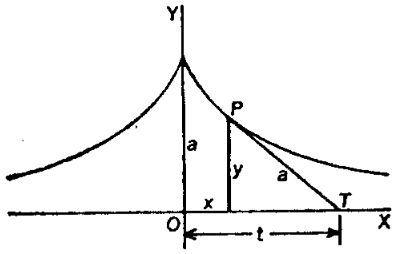
$$[P'A = AP = OA]$$

Tangent curve



$y = \tan x$

Tractrix, Pursuit curve

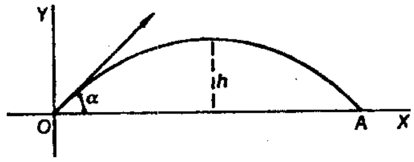


$$x = a \operatorname{sech}^{-1}(y/a) - \sqrt{a^2 - y^2}$$

$$\begin{cases} x = t - a \tanh(t/a) \\ y = a \operatorname{sech}(t/a) \end{cases}$$

$$[PT = a]$$

Trajectory (a parabola)



$$y = x \tan \alpha - gx^2/(2v_0^2 \cos^2 \alpha)$$

$$x = (v_0 \cos \alpha)t$$

$$y = (v_0 \sin \alpha)t - gt^2/2$$

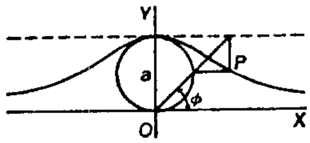
Trigonometric functions

See: Cosecant curve; Cosine curve; Cotangent curve; Secant curve; Sine curve; Tangent curve

Trisectrix

See: Limaçon of Pascal (1)

Witch of Agnesi

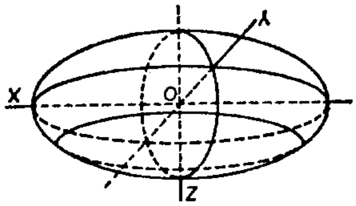


$$y = a^3/(x^2 + a^2)$$

$$\begin{cases} x = a \cot \phi \\ y = a \sin^2 \phi \end{cases}$$

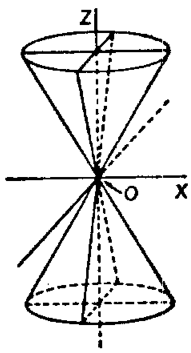
*QUADRIC SURFACES

Ellipsoid



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Elliptic cone

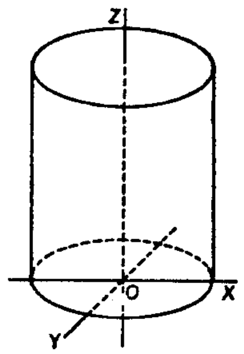


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

*Each of the equations is given for the case where the origin is located at (0, 0, 0), the center of the quadric surface. If, however, the center of the surface is at (h, k, l), replace x by $x - h$, y by $y - k$, and z by $z - l$, and the particular standardized form will be that of the surface with center at (h, k, l). For example, the elliptic paraboloid would be

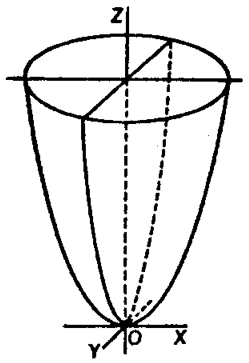
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = c(z - l).$$

Elliptic cylinder



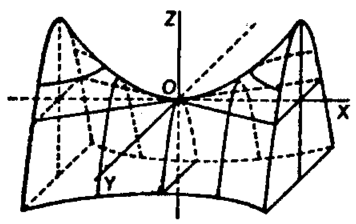
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Elliptic paraboloid



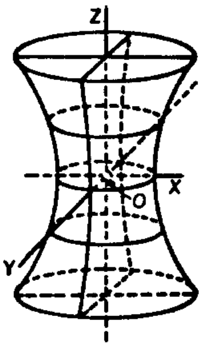
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

Hyperbolic paraboloid



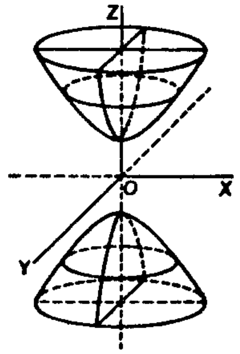
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

Hyperboloid of one sheet



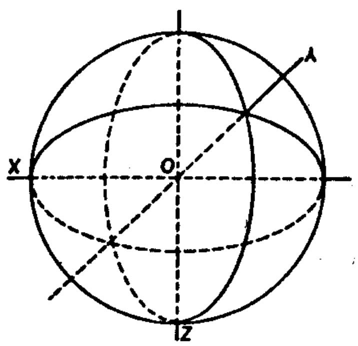
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperboloid of two sheets



$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Sphere



$$x^2 + y^2 + z^2 = a^2$$