ภาคผนวก 1
สูตรอนุพันธ์

สูตรข้างล่างนี้ u, v, w เป็นฟังก์ชันของ x, a, c, n เป็นเลขจำนวนจริง argument ต่าง ๆ ที่อยู่ในฟังก์ชันตรีโกณ วัดเป็นเรเดียน และฟังก์ชันผกผันของฟังก์ชันตรีโกณ และฟังก์ชันไฮเปอร์โบลิกแทนค่าสำคัญ (principal values).

1. $\frac{d}{dx}(a) = 0$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(au) = a\frac{du}{dx}$

4. $\frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$

5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

6. $\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + vw\frac{du}{dx} + uw\frac{dv}{dx}$

7. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{1}{v}\frac{du}{dx} - \frac{u}{v^2}\frac{dv}{dx}$

8. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$

9. $\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$

10. $\frac{d}{dx}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dx}$

11. $\frac{d}{dx}[f(u)] = \frac{d}{du}[f(u)] \cdot \frac{du}{dx}$

12. $\frac{d^2}{dx^2}[f(u)] = \frac{df(u)}{du} \cdot \frac{d^2u}{dx^2} + \frac{d^2f(u)}{du^2} \cdot \left(\frac{du}{dx}\right)^2$

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13. $\frac{d^n}{dx^n}[uv] = \binom{n}{0} v \frac{d^n u}{dx^n} + \binom{n}{1} \frac{dv}{dx} \frac{d^{n-1}u}{dx^{n-1}}$

$+ \binom{n}{2} \frac{d^2 v}{dx^2} \frac{d^{n-2}u}{dx^{n-2}} + \ldots + \binom{n}{k} \frac{d^k v}{dx^k} \frac{d^{n-k}u}{dx^{n-k}}$

$+ \ldots + \binom{n}{n} u \frac{d^n v}{dx^n}$

where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ the binomial coefficient, n non-negative integer and $\binom{n}{0} = 1$

14. $\frac{du}{dx} = \frac{1}{\frac{dx}{du}}$ if $\frac{dx}{du} \neq 0$

* Let $y = f(x)$ and $\frac{dy}{dx} = \frac{d[f(x)]}{dx} = f'(x)$ define respectively a function and its derivative for any value x in their common domain. The differential for the function at such a value x is accordingly defined as

$$dy = d[f(x)] = \frac{dy}{dx}\, dx = \frac{d[f(x)]}{dx}\, dx = f'(x)\, dx$$

Each derivative formula has an associated differential formula. For example, formula 6 above has the differential formula

$$d(uvw) = uv\, dw + vw\, du + uw\, dv$$

15. $\frac{d}{dx}(\log_a u) = (\log_a e) \frac{1}{u} \frac{du}{dx}$

16. $\frac{d}{dx}(\log_e u) = \frac{1}{u} \frac{du}{dx}$

17. $\frac{d}{dx}(a^u) = a^u (\log_e a) \frac{du}{dx}$

18. $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

19. $\frac{d}{dx}(u^v) = vu^{v-1}\frac{du}{dx} + (\log_e u)\, u^v \frac{du}{dx}$

20. $\frac{d}{dx}(\sin u) = \frac{du}{dx}(\cos u)$

21. $\frac{d}{dx}(\cos u) = \frac{du}{-dx}(\sin u)$

22. $\frac{d}{dx}(\tan u) = \frac{du}{dx}(\sec^2 u)$

23. $\frac{d}{dx}(\cot u) = -\frac{du}{dx}(\csc^2 u)$

24. $\frac{d}{dx}(\sec u) = \frac{du}{dx}\sec u \tan u$

25. $\frac{d}{dx}(\csc u) = -\frac{du}{dx}\csc u \cdot \cot u$

26. $\frac{d}{dx}(\text{vers } u) = \frac{du}{dx}\sin u$

27. $\frac{d}{dx}(\text{arc} \sin u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \ (-\frac{\pi}{2} \leqslant \text{arc} \sin u \leqslant \frac{\pi}{2})$

28. $\frac{d}{dx}(\text{arc} \cos u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \ (0 \leqslant \text{arc} \cos u \leqslant \pi)$

29. $\frac{d}{dx}(\text{arc} \tan u) = \frac{1}{1+u^2}\frac{du}{dx}, \ (-\frac{\pi}{2} < \text{arc} \tan u < \frac{\pi}{2})$

30. $\frac{d}{dx}(\text{arc} \cot u) = -\frac{1}{1+u^2}\frac{du}{dx}, \ (0 \leqslant \text{arc} \cot u \leqslant \pi)$

31. $\frac{d}{dx}(\text{arc} \sec u) = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}, \ (0 \leqslant \text{arc} \sec u < \frac{\pi}{2}, \ -\pi \leqslant \text{arc} \sec u < -\frac{\pi}{2})$

32. $\frac{d}{dx}(\text{arc} \csc u) = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}, \ (0 < \text{arc} \csc u \leqslant \frac{\pi}{2}, \ -\pi < \text{arc} \csc u \leqslant -\frac{\pi}{2})$

33. $\frac{d}{dx}(\text{arc vers } u) = \frac{1}{\sqrt{2u-u^2}}\frac{du}{dx}, \ (0 \leqslant \text{arc vers } u \leqslant \pi)$

34. $\frac{d}{dx}(\sinh u) = \frac{du}{dx}(\cosh u)$

35. $\frac{d}{dx}(\cosh u) = \frac{du}{dx}(\sinh u)$

36. $\frac{d}{dx}(\tanh u) = \frac{du}{dx}(\text{sech}^2 u)$

37. $\frac{d}{dx}(\coth u) = -\frac{du}{dx}(\text{csch}^2 u)$

38. $\frac{d}{dx}(\text{sech } u) = -\frac{du}{dx}(\text{sech } u \cdot \tanh u)$

39. $\frac{d}{dx}(\text{csch } u) = -\frac{du}{dx}(\text{csch } u \cdot \coth u)$

40. $\frac{d}{dx}(\sinh^{-1} u) = \frac{d}{dx}[\log(u + \sqrt{u^2 + 1})] = \frac{1}{\sqrt{u^2 + 1}}\frac{du}{dx}$

41. $\frac{d}{dx}(\cosh^{-1} u) = \frac{d}{dx}[\log(u + \sqrt{u^2 - 1}] = \frac{1}{\sqrt{u^2 - 1}}\frac{du}{dx}$, $(u > 1, \cosh^{-1} u > 0)$

42. $\frac{d}{dx}(\tanh^{-1} u) = \frac{d}{dx}[\frac{1}{2}\log\frac{1 + u}{1 - u}] = \frac{1}{1 - u^2}\frac{du}{dx}$, $(u^2 < 1)$

43. $\frac{d}{dx}(\coth^{-1} u) = \frac{d}{dx}\left[\frac{1}{2}\log\frac{u + 1}{u - 1}\right] = \frac{1}{1 - u^2}\frac{du}{dx}$, $(u^2 > 1)$

44. $\frac{d}{dx}(\text{sech}^{-1} u) = \frac{d}{dx}\left[\log\frac{1 + \sqrt{1 - u^2}}{u}\right] = -\frac{1}{u - \sqrt{1 - u^2}}\frac{du}{dx}$, $(0 < u < 1)$

45. $\frac{d}{dx}(\text{csch}^{-1} u) = \frac{d}{dx}\left[\log\frac{1 + \sqrt{1 + u^2}}{u}\right] = -\frac{1}{|u|\sqrt{1 + u^2}}\frac{du}{dx}$